Celebrating Culture through Mathematics

Faculty, Students, and Alumni

Wake Forest University Department of Education

North Carolina Council of Teachers of Mathematics Annual Meeting

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Questions or comments:.

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TOPIC: Navajo Codes in World War II

NCTM STANDARDS: Number and Operations, Communication, Connections, Representation

GOALS:

Students will connect algebraic matrices with the concept of cryptography and its use in Native American culture. They will review basic algebraic concepts of matrices, inverse matrices, and matrix multiplication in the context of coding.

INTRODUCTION:

Navajo code talkers were a vital part of the Allies victory in the battle of Iwo Jima. The complicated language of the Navajo tribe could not be broken by the Japanese. Their unwritten language was used to transmit information on movement on the troops and military tactics. The messages were conveyed as a string of Navajo words that were not correlated. The Navajo code talkers would translate the Navajo word into a comparable English word. Then they used the first letter of the English words to represent the coded word. The code talkers could complete this process for three-line English messages in twenty seconds, but it would take a machine thirty minutes.

ACTIVITIES:

- Part 1. Introduction. Teacher reviews how to construct matrices, how to find the inverse of a matrix, and how to multiply matrices. Then the teacher describes how to convert the letters of the alphabet into numbers. These numbers will be grouped into 2x2 matrices and multiplied by the cipher matrix. This creates the encoded matrices, which are used to construct the encoded message.
- Part 2. Decode a Short Encoded Message. Students will use their knowledge to decode a given matrix as practice.
- Part 3. Create Your Own Encoded Message. Students will create their own cipher matrix, encode a message, and then decode the message.

ASSESSMENT: At the end of the introduction activity, the students will decode a message. They will have to use their knowledge of matrices, inverses of matrices, and matrix multiplication to complete this activity.

Profaca, Mario. (1995). The Navajo Code Talkers. Retrieved September 22, 2006, from: http://mprofaca.cro.net/navajo.html.

Part 1: Introduction

Navajo code talkers were a vital part of the Allies victory in the battle of Iwo Jima. The complicated language of the Navajo tribe could not be broken by the Japanese. Their unwritten language was used to transmit information on movement on the troops and military tactics. One way to create a code is to use matrices. We will use 2x2 matrices for this project.

It is vital to be able to find the inverse of a matrix to use this type of coding technique. To use a graphing calculator for this function, simply enter the values into the 2x2 matrix. These values need to be entered clockwise, starting at the top left corner. Then call up the matrix onto the screen and hit the inverse key ($^{-1}$). This will give you the inverse of the given matrix.

In addition, you must be able to multiply matrices to use this type of coding. To use a graphing calculator for this function, simply enter the values of the two different matrices into two 2X2 matrices. Then multiply the first one by the second one. This will give the matrix product.

To use matrices for cryptography, you have to convert the letters of the alphabet into numbers. The following table tells which letter corresponds to which number:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Once you have the original message that you want to encode, you must translate it into a series of numbers. Then you will group every set of four numbers into a 2x2 matrix, like so: 1 2 3 4 = [1 2] The numbers are inserted into the matrix starting at top
[3 4] left, top right, bottom left, and bottom right.

After creating a 2x2 matrix for every group of four letters (now numbers), you must create a cipher matrix. This is what you will use to encode your message! The numbers do not have to have any sort of significance, other than the fact that the matrix must have a nonzero determinant (so there is an inverse). To find the determinant for the matrix given above, do the following calculation: (1*4) – (2*3). Because this does not equal zero, this could be used as a cipher matrix.

Finally, to encode the message, multiply the cipher matrix by each individual matrix. It is vital that the cipher matrix is the first matrix being multiplied: (cipher) * (group of 4). To decode a message, you must find the inverse of the cipher matrix. Then you will multiply that by each individual encoded matrix. Once again, the inverse of the cipher matrix must be the first matrix being multiplied: (inverse of cipher)*(encoded group of 4).

Part 2: Decode a Short Encoded Message

With the knowledge that you received from the introduction, decode the following message. The cipher matrix for this message is

$$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}.$$

A E I O M T T F A O P B G Q N A

Part 3: Create Your Own Encoded Message

Now it is your turn. Create your own message that you wish to encode. Make sure to use a cipher matrix that has a determinant of one, because then you can find its inverse. It will be easier for you if you choose a message where the number of letters in it is a multiple of four. To earn full credit for this part, show your original message, your cipher matrix, how you used the cipher matrix to encode your matrix, the inverse of your cipher matrix, and how you used the inverse of the cipher matrix to convert the encoded message back to the original message.

Part 4: Assessment

Decode the following message. Its cipher matrix is

$$\begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$$

F Y F B R K C D N K I H V Q E D I V R B N B M E W E K J Q O H K M H L I U V P W A N E S S B T W S E I L E E E X

Show all of your work.

TEACHER NOTES/SOLUTIONS

Part 1: The teacher should use the notes provided in the students' introduction to review that material.

Part 2: The teacher should guide the students through this example.
To find the answer to this problem, first find the inverse of the cipher matrix. The inverse of this matrix is:

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

Then, they should find which numbers correspond to the given letters in the encoded message.

A	**E**	**I**	**O**	**M**	**T**	**T**	**F**
1	5	9	15	13	20	20	6

A	**O**	**P**	**B**	**G**	**Q**	**N**	**A**
1	15	16	2	7	17	14	1

Next, they should create four 2x2 matrices.

$$\begin{bmatrix} 1 & 5 \\ 9 & 15 \end{bmatrix} \quad \begin{bmatrix} 13 & 20 \\ 20 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 15 \\ 16 & 2 \end{bmatrix} \quad \begin{bmatrix} 7 & 17 \\ 14 & 1 \end{bmatrix}$$

Finally, they should multiply the inverse of the cipher matrix by each of these new four matrices.

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 5 \\ 9 & 15 \end{bmatrix} = \begin{bmatrix} 21 & 19 \\ 5 & 13 \end{bmatrix} \ ***$$

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 5 \\ 9 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 20 \\ 8 & 20 \end{bmatrix} \ ***$$

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 5 \\ 9 & 15 \end{bmatrix} = \begin{bmatrix} 15 & 23 \\ 9 & 14 \end{bmatrix} \ ***$$

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 5 \\ 9 & 15 \end{bmatrix} = \begin{bmatrix} 23 & 1 \\ 18 & 19 \end{bmatrix} \ ***$$

***(once converted to a 1-26 number-do this by adding or subtracting 26 until the number falls between 1-16)

21	19	5	13	1	20	8	20
U	**S**	**E**	**M**	**A**	**T**	**H**	**T**

15	23	9	14	23	1	18	19
O	**W**	**I**	**N**	**W**	**A**	**R**	**S**

Use math to win wars.

Part 3: In order to review this part, the teacher should follow each student's process. This will not be for a grade, but rather for extra practice. The teacher will check to see if the following procedures have been completed accurately:

1) The student should use a cipher matrix that has an inverse.
2) The original message correctly has been converted into numbers 1-26 and divided up into 2x2 matrices.
3) These matrices have been multiplied by the cipher matrix to find the encoded message.
4) The encoded matrix has been multiplied by the inverse of the cipher matrix to find the original message.

If the teacher prefers, the students can work in pairs and try to encode and decipher their partner's work. Either way, this is for more practice and not a grade.

Part 4:

The students should first create the corresponding 2x2 matrices for the given encoded message.

$\begin{bmatrix}6 & 25\\ 6 & 2\end{bmatrix}$	$\begin{bmatrix}18 & 11\\ 3 & 4\end{bmatrix}$	$\begin{bmatrix}14 & 11\\ 9 & 8\end{bmatrix}$	$\begin{bmatrix}22 & 17\\ 5 & 4\end{bmatrix}$
$\begin{bmatrix}9 & 22\\ 18 & 2\end{bmatrix}$	$\begin{bmatrix}14 & 2\\ 13 & 5\end{bmatrix}$	$\begin{bmatrix}23 & 5\\ 11 & 10\end{bmatrix}$	$\begin{bmatrix}17 & 15\\ 8 & 11\end{bmatrix}$
$\begin{bmatrix}13 & 8\\ 12 & 9\end{bmatrix}$	$\begin{bmatrix}21 & 22\\ 16 & 23\end{bmatrix}$	$\begin{bmatrix}1 & 14\\ 5 & 19\end{bmatrix}$	$\begin{bmatrix}19 & 2\\ 20 & 23\end{bmatrix}$
$\begin{bmatrix}19 & 5\\ 9 & 12\end{bmatrix}$	$\begin{bmatrix}5 & 5\\ 5 & 24\end{bmatrix}$		

Then the students should multiply each of these matrices by the inverse of the cipher matrix.

$\begin{bmatrix}3 & -4\\ -5 & 7\end{bmatrix}$ * (each of the matrices listed above)

This will create the following matrices, which correspond to letters in the original message.

$\begin{bmatrix}14 & 1\\ 22 & 1\end{bmatrix}$ $\begin{bmatrix}10 & 15\\ 3 & 15\end{bmatrix}$ $\begin{bmatrix}4 & 5\\ 19 & 1\end{bmatrix}$ $\begin{bmatrix}18 & 5\\ 21 & 19\end{bmatrix}$

$\begin{bmatrix}5 & 6\\ 21 & 12\end{bmatrix}$ $\begin{bmatrix}20 & 8\\ 5 & 25\end{bmatrix}$ $\begin{bmatrix}23 & 5\\ 18 & 5\end{bmatrix}$ $\begin{bmatrix}21 & 19\\ 5 & 4\end{bmatrix}$

$\begin{bmatrix}9 & 14\\ 23 & 15\end{bmatrix}$ $\begin{bmatrix}18 & 12\\ 4 & 23\end{bmatrix}$ $\begin{bmatrix}1 & 18\\ 20 & 23\end{bmatrix}$ $\begin{bmatrix}5 & 2\\ 25 & 1\end{bmatrix}$

$\begin{bmatrix}13 & 5\\ 18 & 9\end{bmatrix}$ $\begin{bmatrix}3 & 1\\ 14 & 19\end{bmatrix}$

Finally, the students should have determined which letters these numbers corresponded to in order to find the original message.

14	1	22	1	10	15	3	15	4	5	19	1	18	5
N	A	V	A	J	O	C	O	D	E	S	A	R	E

21	19	5	6	21	12	20	8	5	25	23	5	18	5
U	S	E	F	U	L	T	H	E	Y	W	E	R	E

21	19	5	4	9	14	23	15	18	12	4	23	1	18
U	S	E	D	I	N	W	O	R	L	D	W	A	R

20	23	15	2	25	1	13	5	18	9	3	1	14	19
T	W	O	B	Y	A	M	E	R	I	C	A	N	S

Navajo codes are useful. They were used in World War Two by Americans.

Rubric for Assessment:

It is possible to receive 50 points for this assignment.

Number of Points	
15	Correct matrices for the original message.**
5	Correct inverse of cipher matrix
15	Correct matrices received from multiplying the cipher matrix by the original message.**
15	Correct final message.**

**Give partial credit if so desired. Give one point for each correct matrix, as well as one point for attempting the problem.

Alisa Mook Wake Forest University

TOPIC: Geometry, Pi, and Ancient Civilizations

NCTM STANDARDS: Geometry, Problem Solving, Connections

GOALS:

- Students will use the Internet to complete a guided Pi WebQuest.
- Students will use geometry to study the ways in which ancient civilizations (in Egypt, Babylonia, and Greece) approximated the constant ratio of the circumference of any circle divided by its diameter, which we today call pi (π). They will also learn about the methods used by people in China and India.
- Students will review geometry concepts including the radius, diameter, circumference, and area of a circle, as well as the perimeter of a polygon.

INTRODUCTION:

Pi is a very important ratio in mathematics, representing the circumference of any circle divided by its diameter. Today, we have historical evidence that many ancient civilizations were familiar with this ratio (although they did not call it pi) and that different civilizations used slightly different approaches for calculating pi with varying degrees of accuracy. So what were the ancient equivalents for pi? Let's find out!

ACTIVITIES:

- Part 1. Introduction. Students use the Internet to complete a guided Pi WebQuest.
- Part 2. Approximating Pi. Students work together on geometric constructions/word problems (Ancient Pi Discovery Worksheet) to find an approximation of pi corresponding to each of three ancient civilizations.

ASSESSMENT/PROJECT:

After completing the activities above, students will work with a partner to create a cartoon, poster, song, story, or other approved medium to honor pi and the ancient civilizations we recognize for first using approximations of pi. All final products should also include a definition of pi.

RESOURCES:

Lumpkin, B. (1997). *Geometry activities from many cultures*. Portland: Walch Publishing.

Solli, A. (2005). A chronological history of pi with development activities in problem solving. Retrieved September 21, 2006, from Yale-New Haven Teachers Institute Website: http://www.cis.yale.edu/ynhti/curriculum/units/1980/7/80.07.11.x.html

Name: ______________ **Date:** _______________

Pi WebQuest

Answer each of the following questions about pi after visiting the websites which have been listed to help you with your quest.

π **Visit www.merriam-webster.com and search for pi in the online dictionary.**

1. Pi is the ratio of the ___________ of a circle to its ___________.
2. What is the value of pi rounded to eight decimal places? ___________

π **Go to www.facade.com/legacy/amiinpi/ and enter your birthday (MMDDYY).**

3. When is your birthday? ___________
4. Where in pi does your birthday first occur? ___________

π **Go to http://mathforum.org/t2t/faq/faq.pi.html and read about Pi Day.**

5. When is Pi Day celebrated? ___________
6. Why was this date chosen? ___________

π **Look at the formulas listed on the following webpage:**

http://www.wef.org/ScienceTechnologyResources/TechnicalInformation/ConversionConstantsChemicalInformationFormulas/basic_math.htm.

7. Record the two formulas which involve pi. ___________ ___________
8. Why *don't* the other formulas on this page involve pi? ___________

π **Read the article found at http://www.roanoke.com/news/roanoke/wb/wb/xp-69304 about Gaurav Raja, a high school boy who broke the North American and United States records for memorizing and reciting digits of pi.**

9. How many digits of pi did Gaurav memorize and recite? ___________

π **Read about rational and irrational numbers at**

http://mathforum.org/dr.math/faq/faq.integers.html.

10. Is pi a rational or an irrational number? ___________

Provide a convincing argument for your answer.

____

____

Name(s): ____________________

Ancient Pi Discovery Worksheet

Work through the problems below in order to find an approximation of pi corresponding to each of the following five ancient civilizations.

__________ 1) Ancient Egyptians (1850 BC) used sophisticated mathematical concepts in their daily lives. For example, mathematics was very important for the Egyptians as they worked to build pyramids (which are recognized today for their incredible precision).

An Egyptian scribe named Ahmes (also known as Ahmose) is famous for his mathematical work, especially his work on the Rhind Mathematical Papyrus, a set of complex arithmetic and geometry problems. Ahmes is credited for writing the oldest known text which approximates pi, and the Egyptians will forever be remembered for their rule for finding the area of a circle.

According to Ahmes, the area of a circle with a diameter of 9 units is the same as the area of a square with a side of 8 units. Using our modern formulas for area, find the Egyptian approximation of pi.

__________ 2) In Old Babylonia (1800 BC), clay tablets were used for recording mathematics. Much like the ancient Egyptians, the people in Babylonia (for location, think modern-day Iraq) used complex mathematics to understand and enrich their lives. The Babylonians had an advanced number system, and they used mathematical concepts in the digging and maintenance of canals used for irrigation and transportation.

Use the following to calculate an Old Babylonian approximation of pi (and note that this method *may* have been used in Babylonia):
-Use a compass to draw a circle with a radius of 6 cm.
-Record the length of the diameter.
-Keep your compass open at a width of 6 cm and mark off **six** consecutive arcs on the circle. Connect the points on the circle to create an inscribed **hexagon**.
-Divide the perimeter of the hexagon that you drew by the diameter of your circle.

__________ 3) Archimedes of Syracuse is one of the most well-known mathematicians of ancient Greece (240 BC) and of all time. Some of his important mathematical contributions are related to circles and an approximation for the ratio of the circumference to the diameter of a circle – pi! Other important math concepts attributed to ancient Greece deal with topics in geometry, conic sections, number theory, logical deduction, and formal proofs.

Archimedes began his approximation of pi by stating that the area of a circle must be somewhere in between the areas of its inscribed and circumscribed polygons. Consider a circle with a radius of 1 unit. Use this radius measure to calculate the area of an inscribed hexagon (inside the circle). **Hint: Use your knowledge of special triangles!** This is your lower bound. Now, calculate the area of a circumscribed hexagon (outside the circle). This is your upper bound. Now state the range in which the area of our circle must lie. Why is this range also a range for pi? **Hint: Don't forget the radius measure!** Keep in mind that this procedure can be repeated using polygons with more sides than a hexagon for a more precise answer.

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* In China (260 AD), Liu Hui approximated pi ($\pi \approx 3.14159$) by repeating Archimedes' procedure using a polygon with 3072 sides. Can you imagine?!

* In India (500 AD), Aryabhata approximated pi ($\pi \approx 3.1416$) and may have been one of the first people to recognize that pi is an irrational number.

Student Project Guide

Your assignment is to work with a partner to create a cartoon, poster, song, story, or other approved medium to honor pi and the ancient civilizations we recognize for first using approximations of pi. All final products should also include a definition of pi.

Example: Create a poster which models a timeline for the development of pi throughout history.

You will be graded using the following rubric:

	High Level	Acceptable Level	Low Level
Accuracy	5 4 Students communicate their knowledge of pi and its history clearly/effectively and make few or no errors in content.	3 2 Students do not make incorrect statements about pi or its history but fail to convey a thorough understanding.	1 0 Students fail to define pi accurately and/or make blatantly incorrect statements about its historical significance.
Creativity	5 4 Students use an approved medium to capture an audience's attention and convey important information.	3 2 Students fail to capture an audience's full attention due to distracting use of medium or lack of overall direction.	1 0 Students struggle to apply their medium to the task at hand, use an unapproved medium, and/or fail to focus on the topic itself.
Overall Presentation	5 4 Students use their chosen medium as a means by which to express knowledge and understanding of pi and its history, meanwhile honoring the work of ancient civilizations.	3 2 Students are somewhat clear and effective in their attempts to honor pi and the mathematical works/culture of the studied ancient civilizations.	1 0 Students are not clear and/or effective in their overall effort to define and honor pi and its history through a creative medium. Work is not neat or accurate.

TEACHER NOTES/SOLUTIONS:

Part 1 (Pi WebQuest) Answers:

1. **circumference; diameter**
2. **3.14159265**
3. **answers will vary**
4. **answers will vary**
5. **March 14th (3/14)**
6. **3.14 is a common approximation of pi**
7. **area of a circle = πr^2 and volume of a cylinder = $\pi r^2 h$**
8. **the other formulas involve non-circular shapes**
9. **10,980**
10. **irrational; it cannot be expressed as a ratio of two integers, it has no exact decimal equivalent, etc.**

Part 2 (Ancient Pi Discovery Worksheet) Answers:

1. **$8^2 = \pi*(9/2)^2 \rightarrow 64 = \pi*(4.5)^2 \rightarrow 3.160493827 \approx \pi$**
2. **$(6*6)/(6*2) = 3 \rightarrow 3 \approx \pi$**
3. **area inscribed hexagon = lower bound = $6*(1/2)*b*h = 6*(1/2)*1*(\sqrt{3}/2) \approx 2.598$**

   area circumscribed hexagon = upper bound = $6*(1/2)*(2/\sqrt{3})*1 \approx 3.464$

   since r = 1 and area circle = πr^2, we get area circle = π

***Note: See http://www.ima.umn.edu/~arnold/graphics.html for pictures!**

Katie Simpler Wake Forest University

TOPIC: Geometry, Islamic Art and Culture

NCTM STANDARDS: Geometry, Patterns, Measurement

GOALS:

Students will develop an understanding of shapes and patterns through Islamic art and culture. They will examine basic geometric concepts of symmetry and area, as well as develop their skills with a compass and straight edge.

INTRODUCTION:

Islamic art is centered on geometric designs. It has been used to decorate various facades of artifacts, and is most notable in architecture. A small number of repeated geometric elements combine to create these patterns. Through different colors and shading, artists construct a foreground and a background within the motif. In addition, the repetition makes them seem infinite because the designs are not intended to be contained by the structure that they adorn. One of these patterns, which we will study in depth, is made by five overlapping circles.

ACTIVITIES:

- <u>Part 1. Introduction.</u> Students use the internet to research the Islamic religion and the reason for their use of geometry in their art. They take notes on their findings and share them within groups. The main ideas are briefly outlined and summarized as a class.
- <u>Part 2. Symmetry and Area</u>. Students use a compass and straight edge to draw a pattern template. They analyze the shape's area and symmetries, and use this template to determine the number of figures needed to border one edge of a ten meter wall.
- <u>Part 3. Geometric Figures.</u> Using the traditional methods of Islamic art, students create geometric shapes within the circles of the pattern template. Students will also develop their own methods of creating geometric shapes not necessarily found in traditional Islamic art.

ASSESSMENT: After studying traditional Islamic art, students create their own geometric pattern, decorating it on construction paper and analyzing its symmetry in an attached paragraph.

Fukushimo, C. & Hens, M. (2004). *Islamic Art and Geometric Design: Activities for Learning.* New York: Metropolitan Museum of Art.

Part 1. Introduction.

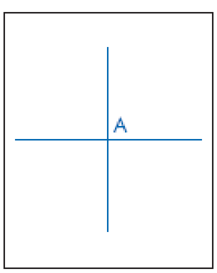
Islamic art and decoration is limited by the religion. Research the history of the geometric patterns in art using the following websites, and others if you choose. Specifically, look for patterns using overlapping circles. What artifacts do they adorn? How are the circles used to distinguish other shapes? How does the coloring and shading change how you view the shapes? You will discuss your findings in groups, and we will summarize them together as a class.

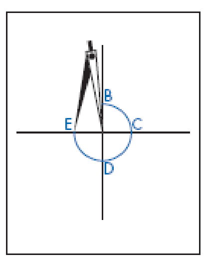
http://www.salaam.co.uk/themeofthemonth/march02_index.php?l=3
http://www.metmuseum.org/toah/hd/geom/hd_geom.htm

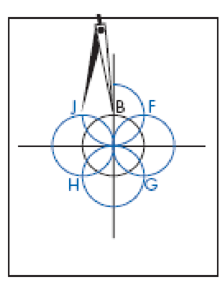
Part 2. Symmetries and Area.

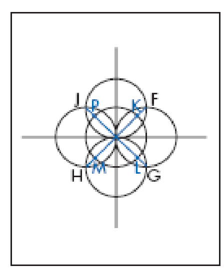
Using a straight edge and a compass, follow the attached instructions to create five overlapping circles. Then answer the questions below.

1. Bisect the page by drawing one horizontal and one perpendicular line. Mark the center as A.
2. Place the compass point at point A and draw a circle. Leave room to draw equal sized circles on each side, at the bottom, and at the top. Mark the points that cross the lines B, C, D, and E.
3. Using points B, C, D and E, draw four more circles. Mark the points where the four circles intersect F, G, H, and J.
4. Use a straightedge to draw the lines FH and JG through the center. These lines intersect the original circle at four equally spaced points at K, L, M, and P.
5. The straight lines both divide the circle into eight equal parts and locate eight equally spaced points—B, C, D, E, K, L, M, P—on the circumference of the original circle. This is the result of the five circles having the same radius.

1.


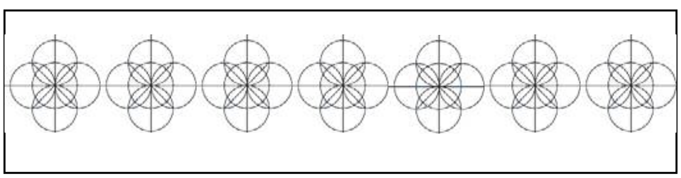
2.


3.


4.


Images: Fukushimo, C. & Hens, M. (2004). *Islamic Art and Geometric Design: Activities for Learning.* New York: Metropolitan Museum of Art.

1. How many reflections can be found in the figure? Sketch and label each line of symmetry.
2. How many rotational symmetries can be found in the figure? List each rotation by degrees.
3. Draw two squares by using lines to connect intersections in the figure.
4. How many triangles can you create from connecting intersections in the figure?
5. What is the diameter of each circle (in centimeters)?
6. Calculate the area of one circle.
7. Approximate the area of the entire figure.
8. Describe how you might add to the figure to complete the flower in the center of the design.
9. Determine how many figures you would need to create a border as pictured on a 10 meter wall.



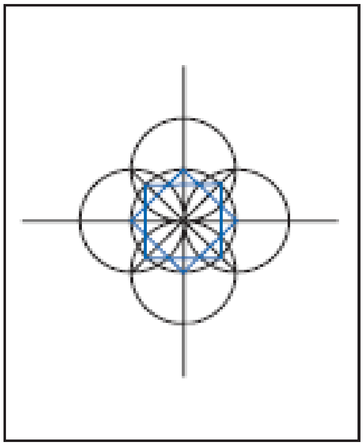
Part 3. Geometric Figures.

Using the five overlapping circles you created in Part 2, find the geometric figures most commonly found in Islamic art. You should copy each shape on a separate sheet of paper. However, you may trace your original five overlapping circles for each subsequent figure. Follow the directions below to sketch each shape, and then look for other ways in which you might form various shapes using the circles.

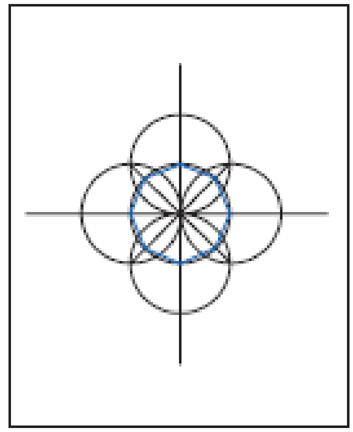
Part 4. Assessment.

Design your own pattern using circles and other geometric shapes formed within them. In a paragraph, describe the symmetries of your design and use the template to create a colorful wall pattern or a border on construction paper.

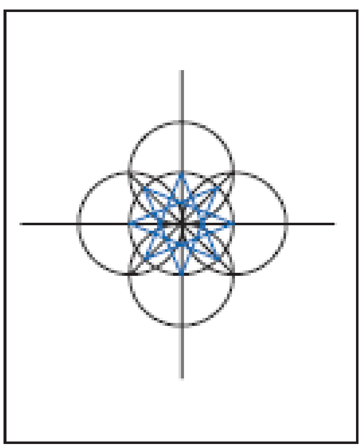
Eight-pointed star:
a) By joining every second point on the original circle, you will create two squares that overlap to form an eight-pointed star.



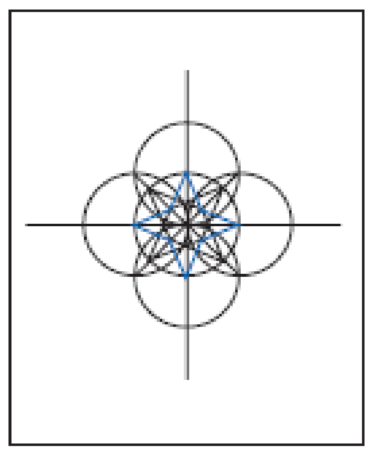
Octagon: To create a regular octagon, use a straightedge to join adjacent points on the circumference of the original circle.



b) By joining every third point, you will create a different eight-pointed star.



Four-pointed star: Embedded in the eight-pointed star (b) is a four-pointed star.



Images: Fukushimo, C. & Hens, M. (2004). *Islamic Art and Geometric Design: Activities for Learning.* New York: Metropolitan Museum of Art.

TEACHER NOTES/SOLUTIONS

Part 1. Lead a discussion allowing students to share what they have researched. Make sure that they understand the important ideas. Show slides of examples of Islamic art. Display the patterns the students created around the room at the end of the activity.

Part 2. 1. 8 lines of symmetry 2. 90°, 180°, 270°, 360° 3. answers may vary 4. answers may vary 5-9. will depend on each student's drawing, Use an example to review how you would find the answers and allow students to correct their mistakes. Then check students' answers after they have completed the activity for a grade.

Example: 5. A student creates the image using circles with diameter equal to 5cm. 6. Area of one circle= 25Π 7. Approximate area of entire image= four circles – one circle (their approximate intersection) = 75 Π 8. One way is to duplicate the four overlapping circles and overlay them on the original image rotated 45° from their original position. This would create four more "petals" in between the pre-existing ones. 9. At a diameter of 10cm each, the design would have to be copied 100 times to border one side of the 10 meter wall.

Part 3. Review traditional shapes to ensure that each student understands the objective. Allow students to share their unique shapes or methods by presenting them to the class.

Part 4. Grade the assessment based on correct analysis of the design and understanding of the general concept expressed in the paragraph.

Rubric for Assessing Design and Paragraph:

Category	Score	Description
No Response	0	Design and analysis paragraph are not attempted, incorrect, irrelevant or do not go beyond restating the prompt.
Minimal	1	Demonstrates only minimal understanding of symmetry. The response is incomplete or contains major errors.
Partial	2	Contains evidence of understanding, but is not well developed. Displays some correct mathematics but may fail to illustrate it clearly in the design.
Satisfactory	3	Demonstrates a clear understanding of symmetry. Generally well developed and well presented, but contains some omissions or minor errors.
Excellent	4	Demonstrates a complete understanding of symmetry and how it is applied to geometric art. Methods for finding and creating symmetries are appropriate and fully developed. Response is logically sound, clearly written and has no significant errors. Design clearly illustrates the mathematical concepts studied.

Justin Allman West Forsyth High School

History of Math Lesson

Objectives: students should be able to

- explain the mathematical implications of the Ishango bone.
- report on where, when, and by whom the Ishango bone was used.

Activity

Students should be taken to the computer lab so that they can utilize web resources for research on the Ishango bone.

Students should visit the following websites:

- http://www.naturalsciences.be/expo/ishango/en/index.html -- this is the site developed by the Belgian Royal Institute of Sciences—the place where the Ishango bone is currently housed.
- http://www.math.buffalo.edu/mad/Ancient-Africa/ishango.html -- this is a site maintained by the math department at the University of Buffalo. It will be the main site for research and contains the most detailed information.

Students should answer the following questions in groups:

(1) Where was the Ishango bone discovered? (you're going to need a more precise answer than simply, "Africa")

(2) What were the people like who lived where the Ishango bone was found?

(3) How old is the Ishango bone? Are there any mathematical objects known to be older than the Ishango bone? If so, where did those objects come from?

(4) How does the bone indicate an understanding of multiplication?

(5) What clues does the bone give that the maker may have had some understanding of a base 10 number system?

(6) What are the sums of the three columns?

(7) The Royal Belgian Institute site says of these sums, "It is hard to believe it is a pure coincidence." Do you agree or disagree. Justify your claim!

(8) Closer looks with a microscope reveal evidence that the Ishango bone was also used as a lunar phase counter. Who, in a hunter-gatherer society, would have use for an object that tabulated the length of our calendar month?

(9) What does this say about who the first "mathematicians" may have been?

Assessment: Each group should report on their findings and opinions of at least one question. For the last 3 questions, the teacher should attempt to stimulate some class discussion on whether or not the rest of the class found the same thing, and whether or not the rest of the class agrees with the assertions of the presenting group.

In a wrap-up of this activity, special emphasis should be placed on the contributions of Africans and women to mathematics throughout history.

John Troutman Southern Durham High School

TOPIC: International Cribs

NCTM STANDARDS: Algebra, Represent and analyze mathematical situations using, algebraic symbols, Use mathematical models to represent and understand quantitative relationships, Geometry, Problems Solving, Connections, Communication, Representation

GOALS:

Students will use the quadratic relationship between a one-dimensional measurement of a traditional housing structure or famous building and its area to construct mathematical statements. The will use these statements to analyze whether the shape they study hold more or less area than a rectangle.

INTRODUCTION:

Ever wondered why so many people today spend most of their time in rectangular rooms? Is this the only shape that people can use to build? Throughout human evolution people have slept, worked, learned, worshipped, and lived in many differently shaped buildings. In this project you will explore one of those shapes, using your research skills and knowledge of quadratics to discover some information about the area of these structures.

ACTIVITIES:

- Part 1. Research. Using links, students research a traditional housing style or architectural structure. After selecting a shape that they would like to use, they follow another set of links to determine a formula for the area and perimeter of this shape.
- Part 2. Determining Dimensions. Once they have determined an area formula for the shape they will be studying, students will be given a desired area and asked to use the formula to determine the length of one or more of the dimensions of this shape. Using the dimension(s) they solved for they will calculate the perimeter of the shape with a fixed area.
- Part 3. Compare. Students will determine the maximum area for a rectangular building of the same perimeter. They will compare the amount of area this shape encloses to the amount enclosed by the shape they studied and write a proposal to a planning committee about which shape to use for a new room at their school.

ASSESSMENT: Students will create a poster that includes all their work along with a picture of the housing structure they studied and the letter they wrote to the planning committee. See grading rubric for more details regarding assessment.

Part 1. Research.

Using the first page of the companion project hand out (attached) and the following links students will research international architectural structures and the formulas used to calculate the area of those shapes.
Some helpful websites are linked at the page on the top of the students' handout. If the link does not work use the list below to give students a variety of options of structures.

Africa
Somalia: **http://www.rit.edu/~africa/somali/somaliAqalPg1.shtml**
Pyramids **http://www.ancientegypt.co.uk/pyramids/home.html**
Maasai **http://maasai-association.org/**
Xhosa **http://www.safari.co.za/africa_xhosa.html**
Kwamsiza **http://www.sahistory.org.za/pages/specialprojects/kwamsiza/menu.html**

Asia
Middle East: **http://en.wikipedia.org/wiki/Islamic_architecture**
Saudi Arabia **http://www.arab.net/saudi/sa_tents.htm**
Mongolia **http://www.chaingang.org/yurtquest/FAQ.html**
Mongolia **http://depts.washington.edu/uwch/silkroad/culture/dwellings/dwellings.html**
Japan **http://www.tjf.or.jp/eng/ge/ge02kutsu.htm**
Japan **http://www.sg.emb-japan.go.jp/JapanAccess/kenchiku.htm**
Kazakhstan **http://intangiblenet.freenet.uz/en/kaz/kaz312.htm**
India **http://www.geocities.com/prashant_iitr/INDEX/pprs/trblar.htm**
China**:** **http://en.wikipedia.org/wiki/Chinese_architecture**

Americas
American Indian **http://www.tipis-tepees-teepees.com/**
American Indian **http://www.kstrom.net/isk/maps/houses/housingmap.html**
Native Americans **http://www.kstrom.net/isk/maps/houses/housingmap.html**
Puerto Rico **http://www.elboricua.com/history.html**
Hopi **http://www.hopi.nsn.us/view_article.asp?id=17&cat=1**
Inuit **http://www.canadianencyclopedia.ca/index.cfm?PgNm=TCE&Params=A1SEC822108**

Europe
Germany **http://www.germanculture.com.ua/library/weekly/german_castles.htm**
England **http://www.royal.gov.uk/output/page563.asp**
France **http://www.chateauversailles.fr/en/**
Rome **http://www.roman-empire.net/society/soc-house.html**
Norway **http://www.olavsrosa.no/en/redaksjonelt.aspx?id=145997**

Math Links (include formulas for most of the shapes):
http://www.math.com/tables/geometry/areas.htm

http://math.about.com/library/blmeasurement.htm

Part 2. Determining Dimension.

Once students have chosen and shape and researched its area formula, they can complete page 1 of the student handout, which asks them to determine dimensions and perimeter for their shape that will give them a specified area.

Part 3. Compare.

Students will then use the perimeter that they just calculated to make a comparison between their shape and a shape that is more commonly used in our culture-the rectangle. Their work in making this comparison will be guided by the instructions on page 2 of the student handout. They might find it helpful to construct a chart similar to the one below to make the information they now have clear.

Shape	Student's shape: the one she/he researched	3 sided rectangle (the other proposal for the arcade)
Area	500 (given)	Y (calculated)
Perimeter	X (calculated)`	X (from part I)
Dimensions	Radius, side length, etc. (calculated)	Length, width (calculated)

Part 4. Assessment.

See attached rubric.

TEACHER NOTES/SOLUTIONS

Part 1. The hardest thing for students to understand was that they were looking for the ground area, so all that mattered about the shape was its base. It will be helpful to emphasize this in pre-research instructions. Also, encourage students to use shapes that may not have an easy area function. While students who choose these shapes may need some additional help to find dimensions, it makes the project more interesting and student products more diverse. Instructors may have to fix some dimensions for some students so that they can perform calculations.

Part 2. Each student should plug 500 into their area formula for the A. Solving this equation for each shape will yield different answers. Make sure that students know to write out all their work and clearly label their answers.

Part 3. Finding the area of the 3 sided rectangle is a maximization problem. Students will create a quadratic function $f(x) = x(P - 2x)$ which gives the area of the rectangle as a function of *x,* its the width (the side that there are 2 of). *P* is the perimeter of the shape (for which they will plug in the value for perimeter that they found in part 1. To find the maximum of area, students will calculate the maximum point of this function (most easily done with a graphing calculator) and take the Y coordinate. This area will almost always be greater, by virtue of the fact that this particular rectangle only requires them to use the fixed perimeter to cover 3 sides. Students' letter to the planning committee should reflect some knowledge of the fact that one shape creates more area given the same amount of building material; however, students should not limit themselves to a purely economic argument. It may be that aesthetics or other factors play a role in their decision leading them to recommend the shape which encloses a smaller area. As long as students explain their reasoning they should be encouraged to make a decision based on whatever factors are most important to them.

Part 4.

Grading Rubric

After completing this project, all students will present their research, mathematical calculations, and conclusions on a poster. The poster should include **all of the components below**. Grades for the project will be assigned based on **the work students show on their poster only**.

Each poster should include:

- a picture of their architectural structure
- a map highlighting the region where their structure is found
- both pages of student handouts
- all mathematical computations
- the memo they wrote for part 2 of the assignment

Student Name: ____

Research on architectural structure (20 points total)
- Choose an appropriate structure from the correct continent (4 points)
- Correctly identify the shape of the structure (2 points)
- Find the correct area formula for this structure (2 points)
- Include important information about the structure's unique characteristics (6 points)
- Include a map of the continent highlighting where the structure is most frequently found (4 points)
- Include a picture of the structure (2 points)

Mathematical computations in part 1 (30 points total)
- Set up an equation to give the dimensions of a model with a 500 ft^2 area (12 points)
- Correctly solve the equation and identify dimensions that would give this area (10 points)
- Correctly determine the perimeter of a figure with that area (8 points)

Mathematical computations in part 2 (30 points total)
- Set up an equation to find the area of the given rectangle with a fixed perimeter (12 points)
- Includes a sketch of the quadratic curve that of this equation with the max point labeled (8 points)
- Correctly identify the maximum area of the rectangular room (10 points)

Concluding memo (20 points total)
- Correctly describes the differences in the two shapes (6 points)
- Correctly identifies the shape which encloses the larger area (6 points)
- Thoroughly explains recommendation to the committee (8 points)

Student's total grade: ________________________

Name: ______________________________

Algebra 2 Project

International Cribs

Visit http://www.wfu.edu/~trouj5/quadraticproject.html and follow the directions on the page to conduct research on an architectural structure in your grade level's continent.

Part 1: Once you have selected a structure for your project and found a formula for determining its area you are ready to complete this project. Follow these steps to discover more about the structure you've chosen.

Step 1: The structure you researched:

____

Its shape: ____

Area formula for this shape: _______________________________________

What are some unique characteristics about this structure? Include information on where it is found, the people who live, work, or worship there, and the types of materials typically used to build it.

____

____

Step 2: Suppose you were told that you needed to create a model of your structure that could enclose 500 ft^2 (approximately the size of a high school classroom). What are some dimensions of your structure that would enclose this area? Show your **all work neatly** on a separate page (hint: use the area formula you wrote in step 1).

Dimensions= ____________________

Step 3: What would the perimeter of your model be if it had these dimensions

P= ______________________

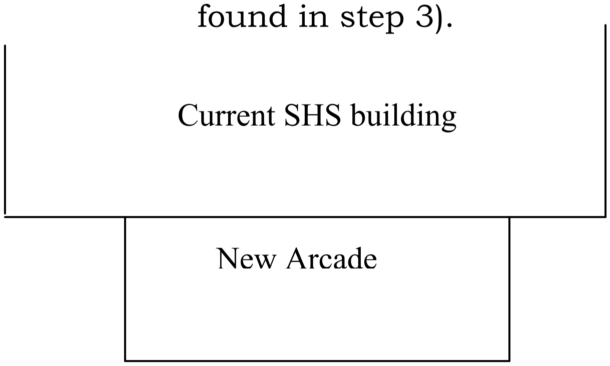
Name: ____________________________

Algebra 2 Grade Level Project:
International Cribs

Part 2: Once you've chosen a structure and calculated some information about its area, consider the following situation at Southern. Use the numbers you discovered above and your knowledge of quadratic equations to answer the following questions and make a recommendation to the planning committee.

Parkland students have decided to build an arcade on the side of the school. Students have suggested two plans for the design of the new arcade.

- **Plan 1**: Build the arcade in the shape of the model you have described above (with the same area, side length, and perimeter).
- **Plan 2**: Build a rectangular arcade using the existing school building as one side (see picture below). The other three sides would have the same perimeter as your model (the number you found in step 3).



Use a graphing calculator to determine the **maximum area** you could enclose with the rectangle in plan 2. Show your work on a separate page. Include a sketch of the quadratic curve you used to answer the question.

Max Area= ____________________

Step 5: Compare the maximum area you calculated in Step 4 to the area of the shape you analyzed in steps 1-3 (area=500 ft^2). Which plan allows for the arcade with the largest area?

On a separate sheet of paper, write a memo to the arcade planning committee about which plan they should use when building the arcade. Include all the following information:

- How the two shapes are different
- Which shape encloses the largest arcade
- A recommendation for which plan to use

Algebra 2 Honors Project

Conic Characters

Directions: You will research a myth or story from your region of the world. You will create a picture of one of the characters from the story using **at least 5 different conic sections**. **Be sure to read the rubric carefully so that you know how your work will be scored!**

Important dates: May 15: We will research stories from your continent. The completed "Student Research Handout" is due the following day (for a classwork grade).
May 19: Rough draft sketch of your Conic Character is due (classwork grade).
May 26: Project due. You will turn in your Student Research Handout, Rubric/Self evaluation, and the final (decorated) version of your Conic Character.

Rough Draft: Sketch of your character with **NO DECORATION**. Each conic should be sketched in a different color. Include the equation (in graphing form) of that conic in the same color. **Equations should be written on a separate sheet of paper.**

Rubric for final copy:

Student Name: ____

Research (20 points)

Criteria (with possible points earned)	Self Evaluation	Teacher Evaluation
Select an appropriate story from your region of the world (2.5 points)		
Correctly identify the area of the culture of the people who wrote the story and the area of the world in which these people lived (2.5 points)		
Identify the main character(s) in the story (5 points)		
Summarize the plot of your story (10 points)		

Math Work (60 points)

Criteria (with possible points earned)	Self Evaluation	Teacher Evaluation
Used at least 5 conic sections to create sketch (5 points)		
All conic are sketch clearly and correctly (25 points)		
All equations are written correctly (25 points)		

Organization and Creativity (20 points)

Criteria (with possible points earned)	Self Evaluation	Teacher Evaluation
Each conic is drawn in a different color and the same color is used to write the corresponding equation; equations are written clearly on a separate sheet of paper. (10 points)		
Drawing looks like the character from the story (10 points)		
Bonus Points: Used creativity and artistic ability to decorate the drawing (up to 15 points will be awarded)		

Student Score:________________

Name: ________________________________

Student Research Handout

Directions: Visit http://www.wfu.edu/~trouj5/glptwo/story_glp_home.html and follow the links to some stories that come from your grade's region of the world. Pick a story and provide the following information about the story you have chosen.

Name of the story: ____

Web address where you found the story: ____

Where does the story come from?
Was the story written by a single author? __________

If so, what was her/his name? __________________ Where did she/he live? ______________

If not, what culture does the story come from? ____________________________

Where did these people live when they wrote the story?

____

Plot. On the lines below, summarize what happens in the story.

____

____

____

____

Main Character. Describe the main character (the one who you will draw).

Name of character: ____

What does she/he look like?: ____

____

____

____

Include any other important characteristics of the character on the lines below:

____

____

Jenny Kropff Wake Forest University

TOPIC: Probability, Igba-ita (Nigerian game)

NCTM STANDARDS: Data Analysis and Probability, Algebra, Connections

GOALS:

- Students will learn the meanings of the mathematical terms: probability, experimental probability, theoretical probability, trial, outcome, frequency, equally likely, and random.
- Students will use a calculator to perform a series of trials using randomly generated numbers.
- Students will relate probability to proportion and ratios through the Nigerian recreational game, Igba-ita.

INTRODUCTION:

Igba-ita is a Nigerian recreation game using their traditional form of currency, cowrie shells, which were often used in the marketplace. The name Igba-ita means "pitch and toss" which is basically what occurs in this game. The game is traditionally played as a type of gambling or betting game played by men, where the players are aiming to win a pot of cowrie shells after a certain number of rounds or trials.

The cowrie shell is a significant symbol all around Africa, not just Nigeria. However, the cowrie shell was used as a form of currency more recently, as well as in ancient times, in Nigeria, while other African countries used the shells as currency throughout ancient times. These shells can mostly be found in the Indian Ocean, and even once were adopted as a form of currency in ancient China, first by using the shells and later making shells of bronze or copper. More recently, these shells have become part of African jewelry.

ACTIVITIES:

- <u>Part 1. Probability lesson introduction.</u> Students will be introduced to important probability terms. They should take notes on these terms so that they can refer to them at a later time, including the other activities.
- <u>Part 2. Calculator Cowrie Shell (or coin) Toss.</u> The students will individually perform the calculator shell/coin toss using their graphing calculators. Using the list and random number generator functions, they can determine long term probabilities of tossing shells/coins. They will also create a scatter plot to be able to visualize the proportion of shells/coins which land a specific way.
- <u>Part 3. Igba-ita game.</u> Students will be in pairs for the Igba-ita activity. Each student will need 4 shell-shaped macaronis (or pennies) and a copy of the rules and score sheet. They will perform 15 rounds of the game and then tally up their scores.

ASSESSMENT:

After completing the activities, the students will be given the *Igba-ita Probability Discovery* worksheet. This worksheet will guide the students through determining probabilities related to ratios of different parts of the Nigerian game, Igba-ita.

Introduction to Probability Scoring Rubric:

Activity	Points per Activity	Points Received
Correct Scatter Plot on Calculator	5	
Igba-ita Score Sheet	5	
Igba-ita Probability Discovery Worksheet	20	
TOTAL POINTS	30	

RESOURCES:

Jackson, I. Cowrie shell history. Retrieved September 22, 2006, from http://www.ifama.com/page7.html.

McIntosh, T. & Carroll, S. (2006). Igba-Ita. Retrieved September 12, 2006, from Queen's University connect-ME family math activities: http://www.educ.queensu.ca/~fmc/december2006/3.htm.

Murdock, J., Kamischke, E., & Kamischke, E. (2002). *Discovering algebra: An investigative approach.* Emeryville: Key Curriculum Press.

WCET, Cincinnati Art Museum, & Association of the Advancement of Art Education. Igba-ita. Retrieved September 21, 2006, from Cincinnati Art Museum's math games at http://www.behindtheglass.org/africaresources/igba.asp.

Part 1. Probability Lesson Introduction Teacher class outline

Have you heard someone use the phrase "chance of" or how likely something is to occur? Where have you heard these phrases (help give examples if none are mentioned- weather, sports, games- lottery, card games, dice games, etc)

The mathematical term for the chance that something specific will happen is called the **probability** of that occurrence.

The probability or chance of something is often given in a percentage, with the percentage being ranging from 0% to 100% (Why can't the probability of something be less than 0% or greater than 100%?). The probability can also be rewritten as a decimal or fraction, where the percent amount is divided by 100% to get the ratio of the chance that something will occur to the entire number of possibilities that could occur (100% of the time).

Or think of the probability as: $\dfrac{\text{\# ways (or times) outcome will occur}}{\text{Total \# of ways (or times) under consideration}}$

which would give you the proportion (or the percent if you multiply that number by 100)

Trial: performing a specific task (one time)
Example- flipping a coin one time

Outcome: result of a trial
Example- when flipping a coin you can either get a head or a tail, so if you flipped a coin and got a head, the outcome of that trial would be a head

Experimental (observed) Probability: collected data from a set amount of trials
Example- We could flip a coin ten times and get HHTHTTTHTT, where we had 4 outcomes of a head and 6 outcomes of a tail. This means that 4 out of 10 times we got a head, or 4/10 = 2/5 of the times we got heads. Then the experimental probability of getting a head was 2/5 = 0.4.

Frequency: number of times a specific result occurs through a series of trials
Example- Like in our experiment where a coin was flipped ten times, the frequency of getting a head would be 4, since we got a head 4 out of the ten times.

Theoretical (actual) Probability: estimated probability based from data
Example- If we were to keep flipping coins for a much larger and larger amount of trials, our experimental probability would become closer and closer to the theoretical probability.

Equally Likely (fair): results are considered equally likely when they have the same theoretical probability of occurring
Example- A penny is generally very evenly weighted with two sides resulting two possible outcomes, heads or tails. Since it is evenly weighted (as long as it has not been damaged or altered to remain fair) it is considered that getting a head is as equally likely as getting a tail.

Random: when the outcome of a trial cannot be predicted
Example- when we flip the coin, we cannot predict beforehand if the outcome will be a head or a tail

Part 2. Calculator Cowrie Shell (or Coin) Toss

Students will use their graphing calculators, preferably with the assistance of a projected calculator by the teacher. The instructions given are based on a TI-83 or TI-83 plus graphing calculator.

We assume that it is equally likely for a cowrie shell to be rolled so that the open side is facing up or the closed side is facing up (theoretical probability of either outcome is .5 or 50% of the time). The calculator can only use numbers, so we will use the number 0 to represent the open side landing upward and the number 1 to represent the closed side landing upward.

Before beginning this activity, make sure the students have the lists (at least 1-4) in their calculators cleared out [STAT, 1:Edit, Highlight L_1 with cursor, CLEAR, ENTER]

Follow the steps to complete the calculator cowrie shell toss:

1. Enter the sequence of integers from 1 to 100 in L_1. These are the numbers of rolls or trial numbers.	1. With the cursor highlighting L_1, 2nd LIST OPS 5:seq(x, x, 1, 100, 1) ENTER
2. Enter 100 randomly generated 0s and 1s into L_2. These numbers designate the outcomes of the rolls.	2. With cursor highlighting L_2, MATH PRB 5:randInt(0,1,100) ENTER
3. Display the cumulative sum of L_2 in L_3. This shows the total number of shells which have landed closed side upward before and including that particular flip.	3. With the cursor highlighting L_3, 2nd LIST OPS 6:cumSum(2nd L_2) ENTER
4. Show the ratio of L_3 to L_1 in L_4. This will display the observed probability of a shell landing closed side upward.	4. With the cursor highlighting L_4, 2nd L_3 ÷ 2nd L_1 ENTER
5. Create a scatter plot using the roll number (L_1) as the x-values and the observed probability of closed side upward (L_4) as the y-values. Set the window appropriately to view the scatter plot.	5. From the main screen, 2nd STATPLOT 1:Plot 1 On ENTER Type: (dots- first choice) Xlist: L_1 Ylist: L_4 WINDOW Xmin = 0 Xmax = 100 Xscl = 1 Ymin = 0 Ymax = 1 Yscl = .1 GRAPH
6. Enter the theoretical probability of rolling a shell to land closed side upward in Y_1 on the Y= screen. Graph this line on the same screen as the scatter plot.	6. Y = Y_1 = .5 GRAPH

Part 3. Igba-ita game

Igba-ita is a Nigerian recreation game using their traditional form of currency, cowrie shells, which were often used in the marketplace. The name Igba-ita means "pitch and toss" which is basically what occurs in this game. The game is traditionally played as a type of gambling or betting game played by men, where the players are aiming to win a pot of cowrie shells after a certain number of rounds or trials.

Below are instructions to play our modified version of Igba-ita:

1. Each player has four cowrie shells to be used in the game.
2. A player must be chosen to go first (this could be done by flipping a shell and one player calling either open or closed, where open and closed are the names for which side is facing up).
3. The chosen player goes first, tossing their four shells on the table.
4. a) If all of the shells are facing the same way, then the player gets a score of 4 and gets to roll again.
   b) If half (2) of the shells are facing one way and the other half (2) another way, then the player gets a score of 2 and gets to roll again.
   c) If anything else is rolled, the player gets a score of 0 for this round and it is now the other player's turn.
5. Continue playing for 15 rolls total, and then add up all of your scores. Person with the largest score wins Igba-ita!

Score Sheet: Keep the tally of your score in the table below
(mark through any rounds you do not play)

Round	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
Total	

Name: ______________________________ Date: ______________________

Igba-ita Probability Discovery

1. When rolling a single shell, what are the possible outcomes?

2. Assuming the shells are equally distributed (the shells are fair) what is the theoretical probability of each of the outcomes when rolling a single shell?

3. If rolling a single shell, what should the observed probability of rolling a shell open side upward over a long period of time be? (think back to the scatter plot)

4. Fill in all of the possible rolls below with an 'O' for open side facing upward or a 'C' for the closed side facing upward:
(*note the series OOCC is different than OCOC)

Shell 1	Shell 2	Shell 3	Shell 4

5. How many total possibilities of shell combinations are there?

6. How many of these times are all shell openings facing upward?

7. How many of the total possibilities include all of the closed sides of the shells facing upward?

8. How many different ways can you get all the shells facing the same way out of all of the possibilities?

9. What is the theoretical probability of getting all of the shells facing the same way in one roll?

10. What is the theoretical probability of half of the shells facing one way and half of the shells facing the other way?

11. What is the probability that a player playing Igba-ita would get to roll the next round as well as the round they are about to roll?

12. What is the probability that a player would get a score of 0 for a round?

13. What was your experimental probability of rolling all shells facing the same way?

14. What was your experimental probability of rolling a second time? (If you rolled three times in a row, then you had two second time rolls)

15. If the theoretical probability of an occurrence was unknown, describe how you could find it.

Name: ______Answer Key____________________ Date: ___________________________

Igba-ita Probability Discovery

(20 points total)

1. When rolling a single shell, what are the possible outcomes? **(1 point)**
   Open side up, closed side up
   Open side up, open side down
2. Assuming the shells are equally distributed (the shells are fair) what is the theoretical probability of each of the outcomes when rolling a single shell? **(1 point)**
   Open side up: ½ or .5
   Closed side up, open side down: ½ or .5
3. If rolling a single shell, what should the observed probability of rolling a shell open side upward over a long period of time be? (think back to the scatter plot) **(1 point)**
   Over a long period of time the observed probability of a shell landing open side upward should be close to ½ or .5.

4. Fill in all of the possible rolls below with an 'O' for open side facing upward or a 'C' for the closed side facing upward: **(4 points)**
   (*note the series OOCC is different than OCOC)

Shell 1	Shell 2	Shell 3	Shell 4
O	O	O	O
O	O	O	C
O	O	C	O
O	C	O	O
C	O	O	O
O	O	C	C
O	C	O	C
C	O	O	C
O	C	C	O
C	O	C	O
C	C	O	O
O	C	C	C
C	O	C	C
C	C	O	C
C	C	C	O
C	C	C	C

5. How many total possibilities of shell combinations are there?
   16 total possible shell combinations **(1 point)**

6. How many of these times are all shell openings facing upward?
   1 time (OOOO) has all of the shell openings facing upward **(1 point)**

7. How many of the total possibilities include all of the closed sides of the shells facing upward?
   1 time (CCCC) has all of the closed sides of the shells facing upward **(1 point)**

8. How many different ways can you get all the shells facing the same way out of all of the possibilities?
   2 ways (OOOO and CCCC) have all of the shells facing the same way **(1 point)**

9. What is the theoretical probability of getting all of the shells facing the same way in one roll?
   The theoretical probability of getting all of the shells facing the same way is 2 ways out of 16 ways which equals 2/16 = 1/8 = .125 **(1 point)**

10. What is the theoretical probability of half of the shells facing one way and half of the shells facing the other way?
    6/16 = 3/8 = .375 **(1 point)**

11. What is the probability that a player playing Igba-ita would get to roll the next round as well as the round they are about to roll?
    8/16 = 1/2 = .5 **(1 point)**

12. What is the probability that a player would get a score of 0 for a round?
    8/16 = 1/2 = .5 **(1 point)**

13. What was your experimental probability of rolling all shells facing the same way?
    Various answers accepted: verify with results of score sheet **(1 point)**

14. What was your experimental probability of rolling a second time? (If you rolled three times in a row, then you had two second time rolls)
    Various answers accepted: verify with results of score sheet **(1 point)**

15. If the theoretical probability of an outcome was unknown, describe how you could find it. Through a very large number of trials, the experimental probability should be a good approximation of the theoretical probability. **(3 points)**

Sarah Greer Wake Forest University

TOPIC: Algebra I and Swedish History

NCTM STANDARDS: Number and Operations, Measurement

GOALS:

Students will learn about the Vasa, a 17^{th} Century Swedish warship, and practice algebra concepts of scale and proportion. They will use measurement skills to construct a model and relate their results to well-known objects.

INTRODUCTION:

In the early 1600s, the Vasa was built according to the order of King Gustavus Adolphus. This Swedish warship was intended to be the greatest warship in the world, but on August 10, 1628 when the ship set sail for the first time, it was an immediate disappointment. The ship sank in the harbor because it did not have enough ballast weight to balance the weight of the guns, upper hull, masts, and sails of the ship. Inquiries were conducted in an attempt to locate the people responsible for the disaster. The final conclusion was that the ship was well built but poorly proportioned.

ACTIVITIES:

- Part 1. Introduction. Students read about the Vasa on the Internet. As a class, they create a list of facts to tell the story of this historical ship. The teacher should supplement this information to complete the story if necessary.
- Part 2. Dimensions and Comparisons. Given the dimensions and scale of the model, students calculate the approximate dimensions of the ship. They will check their values with the exact dimensions available on the Internet and convert the exact dimensions into yards. These dimensions will be compared to a football field to give students a more concrete idea of the size of the ship.
- Part 3. Construction. Students select a scale different from the scale of the model discussed in class. Using this scale they determine the dimensions and create a new model. This model may be 2-dimensional or 3-dimensional and they will decorate their model to relate to what they have learned about Sweden and the Vasa.

ASSESSMENT: When the models are submitted, they will be graded using the attached rubric to assess correct calculations, construction, and design.

Part 1. Introduction.

The Vasa is a famous 17th Century Swedish warship. Explore the following website to learn about the Vasa.

http://www.vasamuseet.se/Vasamuseet/Om.aspx?lang=en

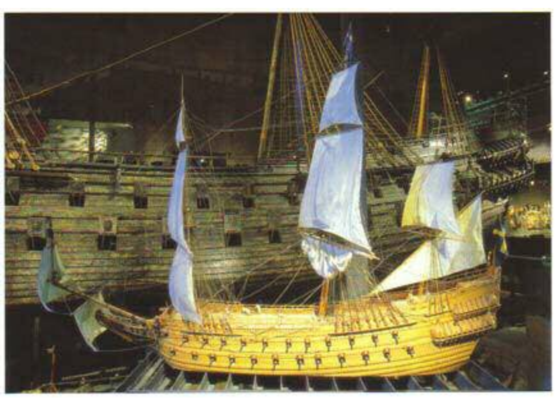
Be prepared to discuss the story of the Vasa. You should be able to answer each of the following questions as part of our discussion.

1. King Gustavus Adolphus was hoping the Vasa would be the mightiest warship in the world. What features were included to make this ship unique?

2. What is the ship most famous for?

3. What caused the disaster?

Part 2. Dimensions and Comparisons.



This model of the Vasa is displayed in the Vasamuseet to show visitors what the ship looked like on the day it sailed. The model is 6.93 meters long and 4.75 meters tall. The scale of the model is 1:10.

1. Use this information to estimate the dimensions of the Vasa.

length:

width:

2. Use the website to find the actual dimensions of the Vasa and convert each measurement from meters into yards (1.093613298 yards = 1 meter).

	in meters	in yards
length		
width		
height		

3. How do the length, width, and height of the ship compare to the length of a football field?

Part 3. Construction.

1. Choose a scale different from the model discussed in Part 2. Using the actual dimensions (length, width, and height) of the Vasa found on the website, calculate the dimensions for a model with your scale.
2. Create your own model of the Vasa. Your model can be 2-dimensional or 3 dimensional and you may choose your materials.
3. Decorate your display using what you have learned about the Vasa. For example, since the Vasa is a Swedish ship your model could be flying a Swedish flag.
4. Attach a 3 by 5 index card to your display. On the card, list your scale and the length, width, and height of your model. You must also turn in a sheet of notebook paper showing your work for calculating the dimensions of your model.

Your model is due on: ___________________________

Be creative!

TEACHER NOTES/SOLUTIONS

Part 1. After students have researched the Vasa on the museum website, lead a class discussion on the history of the Vasa. Create a class list of important information about the ship and its sinking. Supplement the student information if necessary to complete the story.

Part 2.
1. length: 69.30 meters; width: 47.50 meters
2.

	in meters	in yards
length	69.00	75.46
width	11.70	12.80
height	52.50	57.41

3. The length is approximately three-fourths of a football field, the width is slightly more than one-tenth of a football field, and the height is a little more than half of a football field. (Students may have general answers like the sample answer given or specific proportions. The goal is for students to have an idea of how large the ship is by comparing it to something they are familiar with, so general answers are sufficient.)

Part 3.

Provide students with a copy of this rubric with the construction directions.

30	Calculations: Length, width, and height are each worth 10 points. The student must use the selected scale on all three dimensions, show a proportion equation, and correctly complete calculations for each dimension.
30	Model: Length and height are each worth 15 points for a 2-dimensional model. Length, width, and height are each worth 10 points for a 3-dimensional model. The dimensions of the model must match the calculations appropriate for the scale to receive full credit.
20	Display Card: Scale, length, width, and height are each worth 5 points. The card must be displayed on the front of the poster or 3-dimensional display with complete information (including units) to receive full credit.
20	Creativity: Creative use of materials and design related to Sweden and the Vasa are two examples of ways to earn creativity points. It is not necessary to create a scale copy of the Vasa. The model could be a very different looking ship with decoration that gives historical information.
extra credit (5 pts.)	Scale Difficulty: Extra points will be given if your scale cannot be reduced to 1 to some number. For example, a scale of 499:500 would not be a good choice for the project, but it would earn the extra credit points.

Amanda Barnes Wake Forest University

Topic: Math and Latin Music

NCTM Standards: Problem Solving, Connections, Communication, Representation

Goals: Students will connect mathematics to the Latin culture through music. They will review basic mathematical skills such as addition, subtraction, and division in the context of studying notes and rhythms in music. They will also look at the amount of combinations of notes can make up a standard measure of four beats in music.

Introduction: Latin music is an important art form that was developed in Latin American countries, primarily Cuba. This type of music was derived from African religious ceremonies. It is known today for its use in dance music. The strongest characteristic that sets Latin music apart from other music is its rhythm. Various rhythms are played at one time, then other rhythms come in and create an exuberant sound. Most Latin music is created using a rhythmic pattern known as the clave, which involves a 3-2 (occasionally a 2-3) rhythmic pattern. An important characteristic in Latin Music is the "call and response". This involves two main "voices" that call and answer one another within the music. This is done through improvisation between a lead vocalist and instruments.

Activities:

1) At the beginning of class, have websites for students to explore to learn more about Latin Music. If no access to computers is available, use your own computer and guide them through websites using a projector. Students will choose one interesting fact they learned about Latin Music to share with the class, and write down others' facts during the discussion.

2) Have several pieces of Latin music for students to listen to. Have them pick out the rhythm. Can they find the rhythm in those songs? Do they know where the beats fall? Have students listen to you as you give them a beat with drum sticks, or use a metronome found at http://www.metronomeonline.com/. Have students count out several rhythms with you.

3) Show students examples of sheet music found at http://www.8notes.com/. Give students a chart that explains the notes on the page. Have students complete a rhythms worksheet. Have students experiment and write several measures of music in groups.

4) Have students choose one of the measures they created and write that measure on the whiteboard or overhead projector. Also within the four beat measure write down the portion of the measure the note covers. Have the students in each group clap the beats of that measure for the rest of the class.

5) Have students group the types of measures together. For example, list all of the measures that begin with a quarter note, then those that begin with eighth notes, etc. Have a student or the teacher to organize all of the combinations on the board.

6) Discuss any evident patterns in the measures. Have students note how many different ways a measure can be completed if it began with a certain note, such as a quarter note or eighth note.

7) After coming up with all of the possibilities, have students look at an easier problem. Suppose they only looked at two ways to notate a beat and looked at all the combinations in which they could occur in a 4 beat measure. Let's call note A a quarter note and note B two eighth notes. All of the possible combinations are:

AAAA	ABAA	BBBB	BABB
AAAB	ABAB	BBBA	BABA
AABA	ABBA	BBAB	BAAB
AABB	ABBB	BBAA	BAAA

Essentially, there are two types of notes that have to fill up four beats. The number of combinations is $2 \times 2 \times 2 \times 2 = 16$, or $2^4 = 16$

8) Now that we have found this pattern, have students test out several other possibilities in class to see if this formula will work for several cases. If so, come up with a specific formula to use in all situations of music.

Assessment

Have students get into groups of two or three. Groups will each be assigned a combination of notes and beats in a measure, and have to figure out the number of combinations with those particular values of notes. All students will record groups' findings with this assignment. A mini project that can be completed over two are three days can also involve using groups of two or three. Groups will create 10 measures of music, all with different rhythms. Groups will create a rhythms worksheet (along with an answer key for the teacher) for the rest of the class using these measures of music they create. Also, each group will play all of the music they create. They may clap the rhythm, sing the rhythm, sing and dance to the rhythm, display their rhythm on a computer, etc. Students will be allowed to show their creativity with creating music.

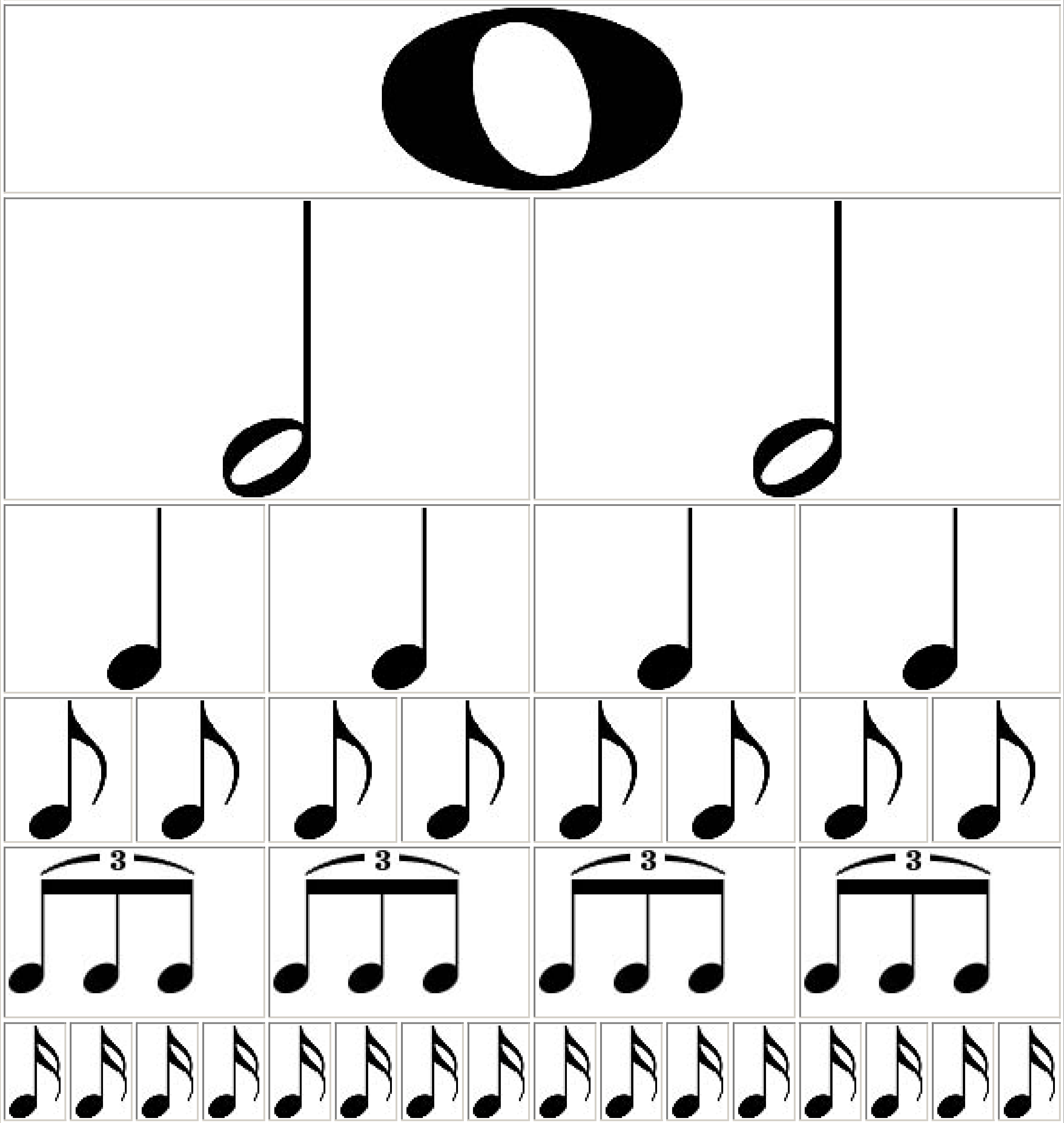
Resources

TeacherVision. (2000-2006). *Jazz and math: Improvisation permutations.* Retrieved 2006, September 4th, from http://www.teachervision.fen.com/musical-notation/lesson-plan/4862.html.

List of ways to notate one beat worksheet: http://www.pbs.org/jazz/classroom/printerfriendlyonebeat.html

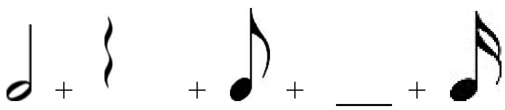
Fraction of a note Chart:
http://www.pbs.org/jazz/classroom/printerfriendlyfractionsworksheet.html
Metronome: http://www.metronomeonline.com/
Samples of Sheet Music: http://www.8notes.com/

Fraction of a Note Chart



Rhythms Worksheet

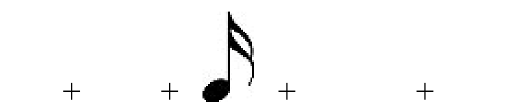
Fill in the missing blanks with fractions or notes or rests. You may use your Fractions of a Note Chart or List of Notations for One Beat Worksheet if you are having trouble. At the bottom of the page for number 5, create your own order of notes that have the values of adding up to one.

1) 

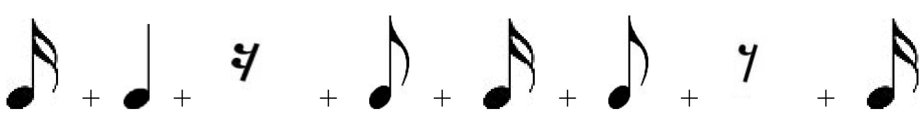
___ ___ ___ 1/16 ____

2) 

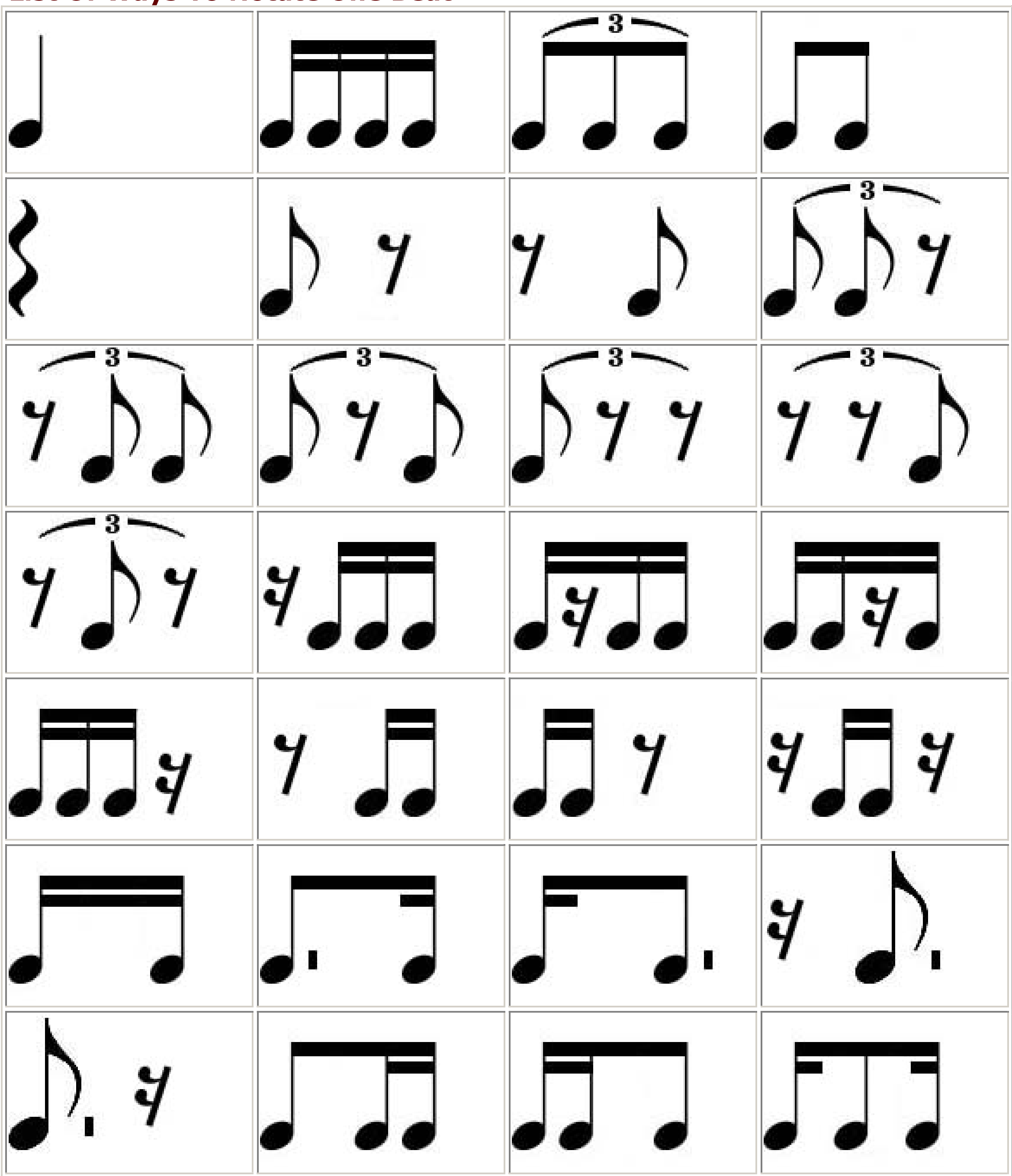
______ ¼ ____ ____ _____

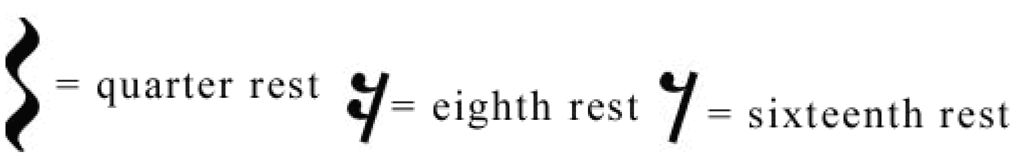
3) 

¼ 1/16 ____ 1/8 1/2

4) 

____ ____ ____ ____ ____ ____ ____ ____

List Of Ways To Notate One Beat

***There is one more way to notate one beat. Can you figure it out?**



Solutions to Rhythms Worksheet:

1) 1/2, 1/4, 1/8, 𝅘𝅥𝅯 or 𝄿 , 1/16
2) 1/4, ♪ or ♩, 1/8, 1/8, ¼
3) ♪ or ♩, 𝅘𝅥𝅯 or 𝄿 , 1/16, ♪ + 𝄾 , 𝅗𝅥
4) 1/16, 1/4, 1/16, 1/8, 1/16, 1/8, 1/8, 1/16

Assessment Rubric: Mini Project

	5	4	3	2	1
10 measures of music	All 10 measures are correct	There are a couple of counting errors within the measures	Students made four or five mistakes within the measures	There are more than 10 mistakes within the 10 measures	Only one or two of the measures are correct
Rhythms Worksheet	Worksheet is neat, has enough space to answer the questions, answer key turned in	Worksheet has a few errors with notation, but neat and organized	Worksheet has errors with notation and fractions, does not have enough space to answer questions	Worksheet has errors with notation and fractions with no space to answer questions, unclear what group members want students to answer	No answer key was turned in, worksheet shows no evidence that group members understand the concepts of notes and their values
Clapping the Rhythm	Group members all participate; all count the rhythms correctly	Group members all participate, few errors were made when counting rhythms	Group members all participate, errors with counting rhythms in more than 7 measures	Not all group members participate; rhythms counted correctly	Not all group members participate; errors were made when counting rhythms
Creativity (up to 5 points extra credit)	Students came up with a new way to display their knowledge of the notes and rhythms		Students were very creative with displaying their knowledge of the notes and rhythms from suggestions given to them in class		Students used a little creativity with the assignment

Leah McCoy Wake Forest University

TOPIC: Geometry, Patchwork Quilts, and Native American Culture

NCTM STANDARDS: Geometry, Measurement, Connections, Communication

GOALS:

Students will connect geometry, patchwork quilts, and Native American culture. They will review basic geometry concepts of area, perimeter, and symmetry in the context of Native American quilts.

INTRODUCTION:

Star quilts are important in Native American culture. Stars are a part of the spiritual tradition in many different tribes. They are believed to be sent by the Great Spirit to watch over the people and to give blessings. Early Indian artifacts frequently included drawings of stars. When Indian women learned the craft of quilting from the pioneer women, star patchwork patterns became favorites. In addition to being used in many homes as bed coverings, star quilts are given as gifts to show respect and honor. They are also used for ceremonial purposes, including births, marriages and deaths. One of the most popular of the Indian star quilts is the Morning Star.

ACTIVITIES:

- Part 1. Introduction. Students research Native American Morning Star Quilts on the Internet. They take notes and discuss the quilts' history and cultural significance. They also construct a paper version of a Morning Star block.
- Part 2. Area and Perimeter. Using the "real-life" Morning Star Block, students determine how many squares would be required to make a full quilt (approximately 80 inches by 80 inches) and sketch the full quilt. Students investigate perimeter and area of each block and of the full quilt.
- Part 3. Symmetry. Students examine their Morning Star Block pattern to determine whether it has one or more lines of symmetry, and whether it has rotational symmetry. They sketch the block and show the symmetry.

ASSESSMENT: At the end of the activity, students write a reflection describing the connections between patchwork quilts, Native American culture, and geometry concepts.

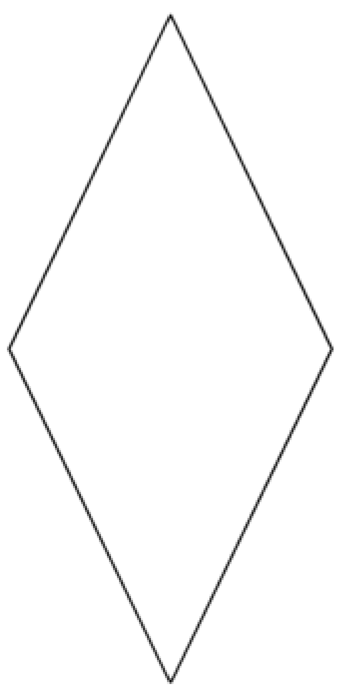
MacDowell, M.L. & Dewhurst, C. K. (1997). *To honor and comfort: Native quilting traditions.* Santa Fe: Museum of New Mexico Press.

Part 1. Introduction.

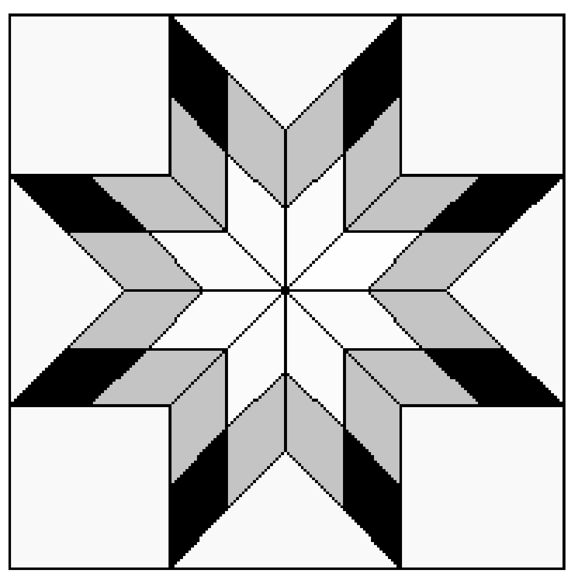
The Morning Star Quilt pattern is significant in Native American culture. Explore the following websites to learn about Morning Star Quilt pattern. You may also do a search for further information. Be prepared to discuss the history of the Morning Star Quilt pattern and its relation to Native American culture.

http://www.bluecloud.org/morningstar.html
http://www.womenfolk.com/quilting_history/nativeam.htm
http://www.native-american-star-quilts.com/

Trace the diamond below and make a pattern of cardboard or heavy paper. Trace around this diamond to make the Morning Star Quilt on large art paper, drawing lines to mark the outside border as below. Color the diamonds to make your own pattern.



Diamond



Morning Star Quilt

Part 2. Area and Perimeter.

Examine the Star Block.

1. What is the measure of each of the sides of your Star Block?
2. What is the perimeter of your Star Block?
3. What is the area of your Star Block?
4. How many squares would you need to make a full quilt that is approximately 80 inches by 80 inches? Sketch it.
5. What is the perimeter of your full quilt?
6. What is the area of your full quilt?
7. What is the relationship of the perimeter of the small block to the perimeter of the full quilt?
8. What is the relationship of the area of the small block to the area of the full quilt?
9. Are these two relationships the same or different? Explain why.

Part 3. Symmetry.

Examine the Star Block.

1. Does your Star Block have line symmetry?
2. How many lines of symmetry does it have?
3. Sketch your block, and draw in all lines of symmetry. Label the type of symmetry.
4. Does your Star Block have rotational symmetry?
5. How many different turns can it make and be symmetric?
6. Sketch your block, and describe its rotations.

Part 4. Assessment.

Write a reflection describing the connections between patchwork quilts, Native American culture, and geometry concepts.

TEACHER NOTES/SOLUTIONS

Part 1. After students have researched this topic on the Internet and taken notes, lead a class discussion on the history and tradition of Native American quilts. Display student-created quilt squares in the classroom, on a bulletin board or other space.

Part 2. 1. 16 in., 2. 64 in., 3. 256 sq. in., 4. 25 It would be 5 x 5, 5. 320 in., 6. 6400 sq. in., 7. 1/5, 8. 1/25

Part 3. 1. yes, 2. 8, 4. yes, 5. 4

Part 4.

Rubric for Assessing Reflection

	Strong	**Adequate**	**Weak**
Introduction	5 4 Topic is introduced in interesting manner, clearly stated. Strong sense of writer's purpose.	3 2 Topic is introduced in somewhat interesting manner, clearly stated. Some sense of writer's purpose.	1 0 Broad, unfocused introduction. Ideas lack interest, originality, and perception. No sense of writer's purpose
Thoughtfulness	5 4 Thoughtful and complete. Student clearly understands both the mathematics and the culture, and clearly connected them.	3 2 Somewhat thoughtful and mostly complete. Student seems to understand both the mathematics and the culture, and made some connections.	1 0 Writing does not reflect thought and may be incomplete. Student lacks understanding of the mathematics and/or the culture, and failed to connect them.
Details and Examples	5 4 Generous use of concrete details and examples.	3 2 Some details and examples.	1 0 No use of details or examples to explain generalizations.
Mechanics	5 4 Writing is clear and effective. No major errors in spelling, punctuation, or grammar. Neatly typed or handwritten.	3 2 Writing is mostly clear and effective. Few errors in spelling, punctuation, or grammar. Neatly typed or handwritten.	1 0 Writing is not clear . Major errors in spelling, punctuation, or grammar. Not neatly typed or handwritten.