Formative assessment is a critical component of effective teaching. By active and continuous collection of data to assess student progress, we can tailor instruction to meet student needs. Creative formative assessment is a key to supporting a growth mindset and maximizing learning. The following are varied mathematics lessons that utilize diverse formative assessments.

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Questions or problems, please contact *Dr. Leah McCoy* mccoy@wfu.edu
There’s No Triangles in Baseball

Chloe Taylor Holt

**Introduction:** This lesson is an investigation into informal proofs of congruency using softball and baseball fields as real-world applications. Students will develop conceptual understanding using and interrelating models of their own creation, knowing and applying facts of congruence, and comparing and contrasting informal proofs of other student groups. Procedural fluency will be developed by student creation of informal proofs of triangle congruence and angle congruence by making conjectures about the fields given some of the fields’ dimensions. Students will also develop reasoning by constructing these informal proofs given a real-world situation and will develop problem solving by testing conjectures concerning the real-world applications. Prior knowledge of rules of congruency will be connected to this lesson in order to come to a solution. If students/groups need support, simplified diagrams of the ball fields may be provided as well as separations of the triangles in question (see Teacher Resources). Students will be representing mathematical information in a visual component, justifying and explaining their reasoning and conclusions to their classmates, comparing their results to the results of other groups, and drawing conclusions from all the information presented throughout the lesson.

**Course:** Math II

**NCSCOS:**
- **NC.M2.G-CO.7:** Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **NC.M2.G-CO.8**: Use congruence in terms of rigid motion. Justify the ASA, SAS, and SSS criteria for triangle congruence. Use criteria for triangle congruence (ASA, SAS, SSS, HL) to determine whether two triangles are congruent.

**Objective**: As a result of this lesson, students will be able to use mathematical reasoning to make conjectures about a real-world situation and construct an argument with a visual model of a conjecture.

**Learning Activities**:
1. Divide students into three groups giving each group one of the informational cards below.

<table>
<thead>
<tr>
<th>Fast-Pitch Softball</th>
</tr>
</thead>
<tbody>
<tr>
<td>A fast-pitch softball field is a square where each <em>vertex</em> is the location of a base. The distance between <em>consecutive</em> bases is 60 feet.</td>
</tr>
<tr>
<td>Important Vocabulary:</td>
</tr>
<tr>
<td>What does vertex mean?</td>
</tr>
<tr>
<td>What does consecutive mean?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slow-Pitch Softball</th>
</tr>
</thead>
<tbody>
<tr>
<td>A slow-pitch softball field is a square where each <em>vertex</em> is the location of a base. The distance between <em>consecutive</em> bases is 65 feet.</td>
</tr>
<tr>
<td>Important Vocabulary:</td>
</tr>
<tr>
<td>What does vertex mean?</td>
</tr>
<tr>
<td>What does consecutive mean?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regulation Baseball</th>
</tr>
</thead>
<tbody>
<tr>
<td>A regulation baseball field is a square where each <em>vertex</em> is the location of a base. The distance between <em>consecutive</em> bases is 90 feet.</td>
</tr>
<tr>
<td>Important Vocabulary:</td>
</tr>
<tr>
<td>What does vertex mean?</td>
</tr>
<tr>
<td>What does consecutive mean?</td>
</tr>
</tbody>
</table>

2. Pose the following two questions to the class:
   - Is the distance from first base to third base the same as the distance from second base to home plate?
   - 2. Is the angle formed between 2nd base, home plate, and 3rd base congruent to the angle formed between 2nd base, home plate, and 1st base?

3. Instruct each group to write statements of their reasoning and create a visual representation on a poster (rubric in Formative Assessment section).
4. Students will then participate in a gallery walk. Each group will have a “docent” to stay with the poster they have created. Groups will rotate around examining other groups’ work and asking the docent questions. Groups will then come back together to discuss and add to their information so the docent may also learn from other groups’ projects. **Note:** It might be helpful either to appoint a student who needs encouragement to participate before the project begins so he/she will be knowledgeable about the work the group is doing OR not tell students this activity will be occurring until right before it starts so they cannot appoint a docent and then let that student do all the work alone.

5. After the gallery walk, students will come together as a class and participate in a Socratic Seminar. Students will arrange themselves to be sitting facing each other (as much as possible). The instructor will begin by asking one of the questions below to initiate discussion that will continue with a series of responses and additional questions. Should the discussion begin to go off-topic or fizzle out, the instructor will ask another of the following questions.
   - Are there similarities in the reasoning between groups?
   - What are some possible reasons the distances between the bases are different for the two different types of softball and baseball?
   - If the fields were a different shape, would you have been able to use the same method you used to solve the problems today?
   - Would knowing the location of the pitcher’s mound when you started the problem have changed your reasoning or process in any way?
   - In what other real-world applications could triangle congruence be used to find solutions to problems?

6. As an Exit Ticket, students will write an individual conclusion to the question, “Why do the different lengths between the bases of the different types of fields not affect the angle measures?”

**Formative Assessments:** Formative assessments will be used in three main forms during this lesson. The poster each group creates and subsequent gallery walk they participate in will be scored by the rubric below. The Socratic Seminar is not formally scored, but serves as a specific time to observe as students discuss with each other what they have learned and ways to apply it. The Exit Ticket is a quick written assignment that will be scored based on inclusion of knowledge and connections made.
Rubric for Field Diagram Questions & Gallery Walk

<table>
<thead>
<tr>
<th>Problem Solving</th>
<th>Reasoning &amp; Proof</th>
<th>Communication</th>
<th>Connections</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task present.</td>
<td>Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.</td>
<td>No awareness of audience or purpose is communicated or Little or no communication of an approach is evident or Everyday, familiar language is used without mathematical language.</td>
<td>No connections are made.</td>
<td>No attempt is made to construct a mathematical representation.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.</td>
<td>Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.</td>
<td>Some awareness of audience or purpose is communicated or Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, or Some formal math language is used, and examples are provided to communicate ideas.</td>
<td>Some attempt to relate the task to other subjects or to own interests and experiences is made.</td>
<td>An attempt is made to construct a mathematical representation to record and communicate problem solving.</td>
</tr>
</tbody>
</table>

This rubric is a slightly modified version of the Exemplars® Classic 5-Level Math Rubric, Copyright ©2001, revised 2012 by Exemplars, Inc. All rights reserved.
<table>
<thead>
<tr>
<th>Problem Solving</th>
<th>Reasoning &amp; Proof</th>
<th>Communication</th>
<th>Connections</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A correct strategy is chosen based on mathematical situation in the task. Evidence of solidifying prior knowledge &amp; applying to the problem solving situation is present.</td>
<td>Arguments are constructed with adequate mathematical basis. A systematic approach &amp;/or justification of correct reasoning is present.</td>
<td>A sense of audience or purpose is communicated &amp;/or Communication of an approach is evident through a methodical, organized, labeled response. Formal math language is used throughout.</td>
<td>Mathematical connections or observations are recognized. Examples are: *clarification of the task *exploration of mathematical phenomenon *noting patterns, structures, &amp; regularities</td>
<td>An appropriate and accurate mathematical representation is constructed and refined to solve problems and portray solutions.</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>An efficient strategy is chosen &amp; progress towards a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, &amp;/or alternate strategies are considered. Evidence of analyzing the situation in mathematical terms, &amp; extending prior knowledge is present.</td>
<td>Deductive arguments are used to justify decisions &amp; may result in formal proofs. Evidence is used to justify &amp; support decisions made and conclusions reached.</td>
<td>A sense of audience &amp; purpose is communicated &amp;/or Communication at the 4 level is achieved, &amp; communication of argument is supported by mathematical properties. Precise mathematical language &amp; notation is used in solution.</td>
<td>Mathematical connections or observations are used to extend the solution to other mathematics or to a deeper understanding of mathematics. Examples are: *testing &amp; accepting or rejecting a conjecture *explanation of phenomenon</td>
<td>An abstract or symbolic mathematical representation is constructed to analyze relationships, and to clarify or interpret phenomenon.</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Teacher Resources:** Below are the simplified diagrams and separate triangles to be used if students/groups need scaffolding. Also below are the formal two-column proofs for the questions posed to the students in Step 2 of the Learning Activities.
<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TS \cong SF \cong FH \cong HT$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle TSF = \angle SFH = \angle FHT = \angle HTS = 90^\circ$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle STH \cong \angle THF$</td>
<td>All right angles are congruent</td>
</tr>
<tr>
<td>$\triangle STH \cong \triangle THF$</td>
<td>SAS</td>
</tr>
<tr>
<td>$\overline{HS} \cong \overline{TF}$</td>
<td>CPCTC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>Reason</th>
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<td>SAS</td>
</tr>
<tr>
<td>$\angle SHT \cong \angle SHF$</td>
<td>CPCTC</td>
</tr>
</tbody>
</table>
El Camino de Santiago – Creating Equations

Kaitlin Yaeger

The Camino de Santiago, or The Way of St. James, is a pilgrimage that those looking for a spiritual journey choose to complete by making their way to Santiago, Galicia, Spain, where St. James is rumored to be buried. The Camino has many starting points depending on how far one is willing to journey, but all starting points eventually lead the traveler through both flatlands and hilly, mountainous terrains. The pilgrimage can be completed on foot or by bike and can lead to any number of experiences along the way. Based on the experiences of previous travelers or by one’s own experiences during the first few days, one can form equations to predict what to prepare for and what will come in the following days.
9-12 Level – Math I - Creating Equations to Represent Scenarios Along the Camino

**Introduction:** Students read scenarios about what happens on the Camino and create equations to predict what would happen to them.

**NCSCOS (Common Core Standards)**
- NC.M1.A-CED.1: Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

**Objectives:**
- SWBAT identify variables, set up linear equations, and solve linear equations both algebraically and graphically based on given scenarios.

**Learning Activities:** Students will learn/review how to:
- Identify and define variable in word problems.
- Set up linear equations from given word problems.
- Solve linear equations algebraically.

Students will learn or review how to identify variables in word problems, set up linear equations from given word problems, and how to solve the linear equations they set up. Students should practice these concepts using the attached worksheet or something similar.

After students have practiced all of these steps, they will be ready to combine them. The formative assessment will show the teacher whether or not the students understood each individual step and how to combine them.

**Formative Assessment:**
The formative assessment will combine everything the students have learned and practiced in these learning activities in the form of a real-life scenario. The teacher should distribute this word problem to students either as a worksheet or projected on the board for students to complete on their own paper. At the end of class, students should turn in their work and the teacher will grade for accuracy on all four of the parts to clarify whether or not each student understands how to identify and define a variable, how to set up a linear equation, how to solve a linear equation algebraically and how to solve a linear equation graphically.
Activity Worksheet – Identifying variables, setting up equations, and solving equations

Student Copy

1. Theo wants to buy a new pair of hiking boots for the Camino de Santiago for $145. He has already saved up $65. He earns $20 for every pool he cleans. How many pools does Theo have to clean in order to have enough money to buy the hiking boots he wants?
   a. State and define the variable: ____________
   b. State the total he is trying to amount to: ____________
   c. Combine the other (non-total) numbers in the problem with the variable: ____________
   d. Set up the equation: _________________
   e. Solve algebraically:

2. Theo is in Europe and needs to buy bottled water to take with him on his hike. Theo buys six bottles of water to take with him on the first day. While at the grocery store, he uses a coupon that took $1.00 off his total bill. If his total came to $11.00, how much does each individual bottle of water cost?
   a. State and define the variable: _________________
   b. State the total he amounted to: ________________
   c. Combine the other (non-total) numbers in the problem with the variable: ________________
   d. Set up the equation: _______________________
   e. Solve algebraically:

3. Theo has heard that in Europe, most restaurants offer a Menu del Dia or a daily special option. The cost of the Menu del Dia is $12. The cost of regular menu items are all the same and all under $12. Theo was really hungry after a long day on the Camino and sat down at a restaurant and ordered one thing from the Menu del Dia and two additional things from the regular menu and his total for everything came to $32. What is the cost of each individual, regular menu item?
   a. State and define the variable: ________________
   b. State the total he amounted to: ________________
   c. Combine the other (non-total) numbers in the problem with the variable: ________________
   d. Set up the equation: _______________________
   e. Solve algebraically:
Activity Worksheet – Identifying variables, setting up equations, and solving equations

Teacher Copy

1. Theo wants to buy a new pair of hiking boots for the Camino de Santiago for $145. He has already saved up $65. He earns $20 for every pool he cleans. How many pools does Theo have to clean in order to have enough money to buy the hiking boots he wants?
   a. State and define the variable: $x=\text{number of pools}$
   b. State the total he is trying to amount to: $\$145$
   c. Combine the other (non-total) numbers in the problem with the variable: $\$20x+\$65$
   d. Set up the equation: $145=20x+65$
   e. Solve algebraically:
      \[
      \begin{align*}
      145 &= 20x + 65 \\
      -65 &= -65 \\
      80 &= 20x \\
      4 &= x 
      \end{align*}
      \]
      Theo has to clean 4 pools in order to afford the new hiking boots.

2. Theo is in Europe and needs to buy bottled water to take with him on his hike. Theo buys six bottles of water to take with him on the first day. While at the grocery store, he uses a coupon that took $1.00 off his total bill. If his total came to $11.00, how much does each individual bottle of water cost?
   a. State and define the variable: $x=\text{cost of each bottle of water}$
   b. State the total he amounted to: $\$11$
   c. Combine the other (non-total) numbers in the problem with the variable: $6x-1$
   d. Set up the equation: $11=6x-1$
   e. Solve algebraically:
      \[
      \begin{align*}
      11 &= 6x - 1 \\
      +1 &= +1 \\
      12 &= 6x \\
      6 &= 6 \\
      2 &= x 
      \end{align*}
      \]
      Each bottle of water costs $2.00.

3. Theo has heard that in Europe, most restaurants offer a Menu del Dia or a daily special option. The cost of the Menu del Dia is $12. The cost of regular menu items are all the same and all under $12. Theo was really hungry after a long day on the Camino and sat down at a restaurant and ordered one thing from the Menu del Dia and two additional
things from the regular menu and his total for everything came to $32. What is the cost of each individual, regular menu item?

a. State and define the variable: \[ x = \text{cost of regular menu items} \]
b. State the total he amounted to: \[ $32 \]
c. Combine the other (non-total) numbers in the problem with the variable: \[ 2x + 12 \]
d. Set up the equation: \[ 32 = 2x + 12 \]
e. Solve algebraically:

\[
\begin{align*}
32 &= 2x + 12 \\
-12 &= -12 \\
20 &= 2x \\
2 &= 2 \\
10 &= x
\end{align*}
\]

Each item on the regular menu costs $10.00.
Identify the variable, set up the linear equation and then solve the equation algebraically according to the following word problem:

During your first two days on the Camino you encounter both flat and mountainous terrain. You’ve found that you can make the most progress when you bike along the flat and downhill parts and walk along the uphill mountainous parts. You have calculated that while walking, you travel at a speed of 2 miles per hour and while biking you travel at a speed of 10 miles per hour. On day 5, the entire trail is downhill or flatland. If, on that day, you plan to travel a distance of 80 miles, how long will it take you to travel that distance?
Identify the variable, set up the linear equation and then solve the equation algebraically and according to the following word problem:

During your first two days on the Camino you encounter both flat and mountainous terrain. You’ve found that you can make the most progress when you bike along the flat and downhill parts and walk along the uphill mountainous parts. You have calculated that while walking, you travel at a speed of 2 miles per hour and while biking you travel at a speed of 10 miles per hour. On day 5, the entire trail is downhill or flatland. If, on that day, you plan to travel a distance of 80 miles, how long will it take you to travel that distance?

Identify the variable: \( x = \text{number of hours}. \)
Set up the equation: \( 80 = 10x \)
Solve algebraically:
\[
\begin{align*}
80 &= 10x \\
\frac{80}{10} &= x \\
8 &= x
\end{align*}
\]
It will take 8 hours to travel a distance of 80 miles.
Colony Coordinates

Julianna Miller

Students become familiar with the coordinate plane through a social studies integrated activity on the Thirteen Colonies. Each student will design a typical town center, plotting the location of buildings using ordered pairs. Students write a brief narrative of their day in the colony, through the perspective of a citizen. They must describe the locations they visited, what they did there, and calculate the distance between the buildings to describe how far they walked.

Fifth grade – NCSCOS

- Math Standards
  - 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
- Social Studies Standards
  - 5.H.1.2 Summarize the political, economic and social aspects of colonial life in the thirteen colonies.
- Writing Standards
  - 5.W.3 Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences.
Objectives

- SWBAT plot ordered pairs on a coordinate system
- SWBAT create a key for their town map using miles
- SWBAT write a historically accurate narrative
- SWBAT write their narrative through the point of view of an individual living in one of the 13 colonies
- SWBAT calculate the number of miles their character traveled

Learning Activities

Students will work in pairs to design a map of an imaginary town center in one of the thirteen original colonies, using a coordinate plane. They will have access to links so they can research what life was like for different people in the colonies. Students will also use these links to learn about the different buildings that made up a town center. In addition to designing a map and plotting the ordered pairs, students will put themselves in the shoes of a citizen and write what their daily life might be like, including where they visited on the town map.

Formative Assessment

The teacher will use student’s maps of their town, the calculated distance they “traveled” and an appropriate key as a formative assessment. The rubric for this is attached. Though students are working in pairs, they each should produce a copy of their map. The teacher will also use the written narrative to formatively assess student understanding of the time period.
Activity Sheet for Teacher Use
Colony Coordinates Activity

Learning Activity:
- Students work in pairs to graph points on paper to design the layout of the town center
- Students then decide what building will be at each point
- Each student produces a map
- Using the links below, students learn about the roles of people in colonial times and choose a character
- Students individually write a narrative, describing where they went on their day in the town center, and what they did at the different locations
- Using their maps, students must create a key to explain how far apart each location is (ex: four boxes is a mile)
- Once the key has been created, students will calculate how far they traveled on their day in the village and include this in their narrative

Links for Research:
- http://www.history.org/Almanack/life/classes.cfm
- http://www.history.org/Almanack/places/index.cfm
Rubric for Graph:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>0-3 Points</th>
<th>4-6 Points</th>
<th>7 Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>Few to no points are correctly labeled</td>
<td>Most points are correctly labeled</td>
<td>All points are correctly labeled</td>
</tr>
<tr>
<td>Key</td>
<td>Student did not choose an appropriate number of miles for the key</td>
<td>Student chose and appropriate number of miles for the key</td>
<td>/7</td>
</tr>
<tr>
<td>Calculation</td>
<td>Calculation of miles is incorrect</td>
<td>Calculation of miles is correct</td>
<td>/7</td>
</tr>
</tbody>
</table>

Total: /21

Rubric for Narrative:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>0-2 Points</th>
<th>3-4 Points</th>
<th>5 Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of Buildings</td>
<td>Student describes use of one building visited or not at all</td>
<td>Student describes historical use of few buildings visited.</td>
<td>Student describes historical use of all buildings visited on map.</td>
</tr>
<tr>
<td>Historical Accuracy in Point of View</td>
<td>Student does not write through appropriate historical point of view.</td>
<td>Student provides some historical accuracy in point of view.</td>
<td>Student writes through historically accurate point of view.</td>
</tr>
<tr>
<td>Social Structure</td>
<td>Writing barely or does not at all accurately portray social standing of colonial citizen through the places traveled</td>
<td>Writing provides some accuracy of social standing of citizen through places traveled.</td>
<td>Writing accurately portrays social standing of colonial citizen through places traveled.</td>
</tr>
</tbody>
</table>

Total: /15
My name is Ms. Hampton and I am the wife of a blacksmith. We are of the middling social class, and provide goods for the wealthier families. Today I had a busy day. In the morning, I stopped by the church to pray, as it was Sunday and all attend service today. Then, I went to the market square to buy vegetables that the local farmers were selling. I went to the post office in the afternoon to send a letter to my cousin, wishing them a merry Christmas. Though it is only November, the mail carriers are so slow that I fear my holiday wishes won’t make it on time. At the end of the day, I returned home to the shops, where my family and I live above our smithery. It was quite a long day! I walked three miles!
Activity Sheet for Student Directions

Colony Coordinates Activity

Today you will design a town center during the 1600’s in one of the original thirteen colonies! You will research the function of different buildings that existed during the time period and plot them as ordered pairs on a graph (like the one below). You will work in pairs and have identical maps as your partner, though each must turn in separate graphs.

Next, you will pretend to be a character from the time period and write a narrative of all the places you traveled today, the buildings you went to, and what you did there. Your narratives will be written individually and will not be part of your pair work. Make sure to write from the perspective of your character and describe the places you visited with historical accuracy. Society in the 1600’s was less inclusive, so remember to read the different roles of the individuals, what buildings they were allowed to visit and what they did there. You can learn all about this by using the resources below!

Finally, you will calculate the distance you traveled. In order to do this, make a key for your map that explains how many squares is equal to one mile. This should be put on your map. Make sure to include this in your narrative.
Here is a student example of a narrative:

My name is Ms. Hampton and I am the wife of a blacksmith. We are of the middling social class, and provide goods for the wealthier families. Today I had a busy day. In the morning, I stopped by the church to pray, as it was Sunday and all attend service today. Then, I went to the market square to buy vegetables that the local farmers were selling. I went to the post office in the afternoon to send a letter to my cousin, wishing them a merry Christmas. Though it is only November, the mail carriers are so slow that I fear my holiday wishes won’t make it on time. At the end of the day, I returned home to the shops, where my family and I live above our smithery. It was quite a long day! I walked three miles!

Resources:

- http://www.history.org/Almanack/life/classes.cfm
- http://www.history.org/Almanack/places/index.cfm
Don’t Fret, We Have Math

Marie Mickley

INTRODUCTION: How do musicians know where to place their fingers on the neck of a guitar in order to produce pleasant sounding notes? Our understanding of the modern notes scale system as we know it today can be accredited to the connection that Pythagoras made between music and math. According to Pythagoras’ observations, by placing the frets of a guitar at certain proportions of the full string, we know where to place our fingers in order to produce notes. Under closer examination, the placement of guitar frets can be represented as a geometric sequence wherein the distance from one fret to the bridge is decreasing by the same ratio.

COURSE: Discrete Mathematics

NCSCOS STANDARD:

- Competency Goal 3: The learner will describe and use recursively-defined relationships to solve problems
  - 3.01 Use recursion to model and solve problems
     - Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas

OBJECTIVES: By the end of this lesson, students will be able to:

- Identify and define a geometric sequence
- Create a recursive formula to represent the elements of a sequence
- Apply the following knowledge to a real-life context

ACTIVITIES:

- The class will review a worksheet about Pythagoras’ investigation of music in a mathematical context and his conclusion that guitar strings can be divided into specific fractions of the full length in order to produce notes
- Students will use technology to fill out a basic labelling of the parts of a guitar
- In small groups, students will use string as a representation of a guitar string and their knowledge on Pythagoras’ proportions for guitars to examine the relationship between distances to the bridge from different frets
- Students will record their findings in a table
- Students will collaborate with group members to find a way to represent the relationship in equation form and present their findings to the groups
- The class will reconvene to create concrete definitions for the found relationship and a recursive formula

ASSESSMENT: Student group presentations will be graded along the guidelines of the attached rubric. Additionally, students will complete individual one-minute essays on what they learned and what questions they still have.
Pythagoras and His Funky Fresh Grooves (Teacher Version)

Though Pythagoras is most famously known for his theorem on right triangles, he also figured out how long guitar strings need to be in order to create certain notes. His calculations are the foundation for the placement of frets on guitars. For instance, pressing down on the 12th fret of a guitar makes the string half of its original length, producing an Octave note.

**Directions:** Use the computer to label the indicated parts of the guitar. (Teachers should review the answers to this diagram on the board before moving on)
Pythagoras and His Funky Fresh Grooves

Name:________________________

Though Pythagoras is most famously known for his theorem on right triangles, he also figured out how long guitar strings need to be in order to create certain notes. His calculations are the foundation for the placement of frets on guitars. For instance, pressing down on the 12th fret of a guitar makes the string half of its original length, producing an Octave note.

Directions: Use the computer to label the indicated parts of the guitar.
String Activity

1. Students should be divided into groups of around four people and each group member should be assigned one of the following roles:
   a. Measurer: This person should be the one using the meter stick to measure the distance from each fret to the bridge
   b. Recorder: This student will record the Measurer’s findings in a table
   c. Speaker: This will be the student that presents the group’s findings to the class
      i. If the class cannot be evenly divided into groups of four, then students can share the Speaker responsibility
   d. Facilitator: This student will make sure that other group members stay on task and act in a timely and efficient manner
2. Each group will cut a length of string at least a foot and half long for themselves, though they need not cut it at a specific length
3. Students should tape down the ends of their string on a flat surface to make measuring more consistent
4. The Measurer measures the full length of the string and the Recorder records this length in the table
5. Group will divide the length of the string into Pythagoras’ ratios using precise measuring techniques (students need only measure out to the 10th fret) and indicate these ratios with a marker mark on the string
6. Students can understand the marks as the frets of the guitar and the bottom of the string is the bridge
7. Measurer then measures the distance from each fret to the bridge order of frets (distance from 1st fret to bridge, distance from 2nd fret to bridge, etc.) and Recorder inputs data into table
8. Students then stop to discuss what trends they notice from one distance to the next. Speaker should be making notes so the he/she can later present thoughts to class as part of presentation
9. Students then complete second column of table by calculating the ratio of consecutive distances, which should all be around .94
10. Prompt to students on bottom of graph worksheet: Create an equation to solve for the distance from the 11th fret to the 10th fret, an equation for the distance from the 12th fret to the bridge, a general equation from the nth fret to the bridge. Students will work towards creating a recursive formula of a geometric sequence. Teacher should look for answers similar to the following: \(a_n = a_{n-1} \cdot (.94)\) where .94 is the common ratio.
11. Students present findings to class with each group speaking ~5 minutes and teacher uses provided rubric.
12. Whole class comes back together to complete the attached notes to provide concrete definitions of investigation.
13. Students complete a one-minute essay in the last couple minutes of class
Table for Fret to Bridge Relations

Group Members:

<table>
<thead>
<tr>
<th>Fret</th>
<th>Distance from fret to bridge</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td></td>
<td></td>
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<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>6&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>8&lt;sup&gt;th&lt;/sup&gt;</td>
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<tr>
<td>9&lt;sup&gt;th&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10&lt;sup&gt;th&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Work should be completed on a separate piece of paper*

- Create an equation to solve for the distance from the 11<sup>th</sup> fret to the 10<sup>th</sup> fret
- Create an equation for the distance from the 12<sup>th</sup> fret to the bridge
- Create a general equation from the nth fret to the bridge

Message to the Speaker: Be prepared to take five minutes to present your group’s findings to the class.
Geometric Sequences (Teacher Version)

The distances you have measured and recorded are an example of a geometric sequence. With that new fact, how would you define a geometric sequence?

- Teacher should look closely for use of vocab words like ratio, multiply, terms, related

Geometric Sequence: (This definition should be provided by the teacher verbally and written on the board) A sequence of numbers where each term after the first is found by multiplying the previous term by a common ratio

- Common Ratio: Fixed, non-zero number
- What was the common ratio for the guitar fret distances? .94

Recursive Formula: (This definition should be provided by the teacher verbally and written on the board) a formula that requires the computation of all previous terms in order to find the value of \( a_n \).

- If we are solving for the distance from the 9th fret to the bridge, \( n=9 \)
- How about from the 12th fret? 12
- Using the general formula created for findings the distance from the nth fret to the bridge, what is the general form of a recursive formula for a geometric sequence? Discuss with partner.

(After some input from class and discussion, teacher should present the following equation: \( a_n = r \cdot a_{n-1} \) where \( r=\)common ratio, \( a_n=\)the nth term, and \( a_{n-1}=\)the n-1 term)

- For the case of finding the distance from the 11th fret to the bridge, fill in the following:
  - \( a_n=\) distance from 11th fret to bridge
  - \( r = .94 \)
  - \( a_{n-1}=\) distance from 10th fret to bridge (will vary according to groups)
Geometric Sequences

Name:

The distances you have measured and recorded are an example of a geometric sequence. With that new fact, how would you define a geometric sequence?

**Geometric Sequence:** A sequence of numbers where each term after the first is found by multiplying the previous term by a **common ratio**

- **Common Ratio:**
- What was the common ratio for the guitar fret distances?

**Recursive Formula:** (This definition should be provided by the teacher verbally and written on the board) a formula that requires the computation of all previous terms in order to find the value of $a_n$.

- If we are solving for the distance from the 9th fret to the bridge, $n=$
- How about from the 12th fret?
- Using the general formula created for finding the distance from the nth fret to the bridge, what is the general form of a recursive formula for a geometric sequence? Discuss with partner.

- For the case of finding the distance from the 11th fret to the bridge, fill in the following:
  - $a_n =$
  - $r =$
  - $a_{n-1} =$
# Guitar Fret String Activity Rubric

**Group Members:**

<table>
<thead>
<tr>
<th></th>
<th>Excellent</th>
<th>Satisfactory</th>
<th>Below Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Leadership in Role</td>
<td>Students exhibited proficiency in role responsibilities</td>
<td>Students put moderate effort into role responsibilities</td>
<td>Students made little to no attempt to fulfill role responsibilities</td>
</tr>
<tr>
<td>Group Cooperativeness</td>
<td>Was open to groupmates’ input and reacted appropriately to differing input</td>
<td>Acknowledged groupmates and communicated moderately</td>
<td>Did not listen to groupmates or communicate with them</td>
</tr>
<tr>
<td>Analysis</td>
<td>Students carefully analyzed the information collected and drew appropriate conclusions supported by evidence. Mathematical reasoning was evident.</td>
<td>Students’ conclusions could be supported by stronger evidence. Level of analysis could have been deeper. Mathematical reasoning was mostly adequate.</td>
<td>Students’ conclusions simply involved restating information. Conclusions were not supported by evidence. Mathematical reasoning was not evident.</td>
</tr>
</tbody>
</table>

**Total:**
Teacher Instruction for One-Minute Essay Assessment

- Teacher should present the following reflection questions on the board for students to answer individually in a one minute time period and turn in before leaving
  - What is the most interesting thing you have learned today?
  - What questions do you still have? If you have no questions, come up with a question to make the topic more challenging.
Every year MLB hosts a home run derby where selected all-star players compete in a home run hitting contest. Leading up to this lesson, students have just finished a unit on parametric equations. They have learned how to write, solve, and graph them. In this lesson, students will apply parametric equations to a real world situation. They will use them to model and solve problems in the context of baseball. Students will be given certain information about a player’s at bat and will have to model that player’s swing using parametric equations to determine if the player hit a home run. They will use information such as the player’s height, the speed of the ball, the angle the ball left the bat, and the distance to the outfield wall. Students will have to model the different situations with equations and graphs.

**NCSCOS Pre-Calculus Objective 2.06**: Use parametric equations to model and solve problems.

**Objectives:**
Students will be able to solve parametric equations given all necessary information. Students will be able to interpret parametric equations in the context of a problem.

Parametric equations define relations as sets of equations. We use parametric equations to trace the movement of an object along a path according to time. We are not only looking for a particular position, but also the direction and time. We use parametric equations because they yield more information. For example, we can use parametric equations to calculate the path a moon follows as it orbits a planet, which simultaneously rotates as the sun.
For this learning activity the class will be split up into two teams. Both teams will take part in a draft where they will select different players to be on their home run derby team. Once the teams have been selected, two players will go against each other at a time. For each player, the students will be given all of the necessary information needed to model the batter’s swing using a parametric equation. Students will have to solve the parametric equation and graph it. Students will then use the information they’ve gathered to determine whether the player hit a home run. If the player hit a home run, they score a point for their team. This process will continue until all players have gone up to bat. The team with the most home runs wins.

During the activity, students will be completing their work on a separate sheet of paper. For each problem corresponding to their team, students will work out the problem, and sketch the graph. Students will turn in this work at the end of class as a formative assessment. This will make sure all students are engaged and working in the group, and will measure each student’s knowledge rather than just measuring the group. The teacher will look over the worksheets and offer feedback to students.
Teacher Information

1. Split the class in half.
2. Have each team come up with a team name.
3. Pass out the student worksheet.
4. Each team will partake in the draft and alternate picking players between the eight available players until all players are chosen. Each team will have four batters.
5. Each team will select a player to begin. They will be given the initial velocity, initial height, and angle of contact for the swing. They will plug that information into the corresponding equation to find at what time the ball reaches the fence. They will then plug the time, with the rest information, into the other equation to determine how high the ball was when it reached the fence.
6. The students will use the height that the ball is at the fence to report whether the player hit a home run. For each home run a player hits the team gets a point.
7. This process continues until all players have hit. The team with the most points wins.
8. Students will turn in their work to be graded as a formative assessment.
Solutions:

The two equations we’re using are \[ x = v_0 \cos \theta \ t \] and \[ y = h_0 + v_0 \sin \theta \ t - 16t^2 \]. For students to determine if it’s a home run, first they need to find what time the ball reaches the fence. To do this they will plug 407 for \( x \) in the first equation, since that’s the distance to the wall, and solve for \( t \). Once they found \( t \) they will then use the second equation to figure out what height the ball is at when it reaches the wall. They are told the wall is 8 feet high, so if the ball is higher than 8 feet when it reaches the wall it is a home run. The detailed solutions are shown below.

**Gincarlo Stanton**- 407 = 100 \( \cos(15) \) \( t \) → \( t = 4.21 \) seconds → \( y = 3 + 100 \sin(15) \) (4.21) − 16(4.21)^2 → \( y = -171.62 \) feet → NO home run

**Gary Sanchez**- 407 = 125 \( \cos(45) \) \( t \) → \( t = 4.60 \) seconds → \( y = 3 + 125 \sin(45) \) (4.60) − 16(4.60)^2 → \( y = 71.03 \) feet → YES home run

**Mike Moustakas**- 407 = 120 \( \cos(35) \) \( t \) → \( t = 4.14 \) seconds → \( y = 2 + 120 \sin(35) \) (4.14) − 16(4.14)^2 → \( y = 12.72 \) feet → YES home run

**Miguel Sano**- 407 = 130 \( \cos(30) \) \( t \) → \( t = 3.62 \) seconds → \( y = 4 + 130 \sin(30) \) (3.62) − 16(3.62)^2 → \( y = 29.63 \) feet → YES home run

**Aaron Judge**- 407 = 152 \( \cos(20) \) \( t \) → \( t = 2.85 \) seconds → \( y = 3 + 152 \sin(20) \) (2.85) − 16(2.85)^2 → \( y = 21.2 \) feet → YES home run

**Justin Bour**- 407 = 115 \( \cos(45) \) \( t \) → \( t = 5.01 \) seconds → \( y = 2 + 115 \sin(45) \) (5.01) − 16(5.01)^2 → \( y = 7.80 \) feet → NO home run

**Cody Bellinger**- 407 = 120 \( \cos(30) \) \( t \) → \( t = 3.92 \) seconds → \( y = 3 + 120 \sin(30) \) (3.92) − 16(3.92)^2 → \( y = -7.66 \) feet → NO home run

**Charlie Blackmon**- 407 = 145 \( \cos(20) \) \( t \) → \( t = 3.00 \) seconds → \( y = 3 + 145 \sin(20) \) (3.00) − 16(3.00)^2 → \( y = 7.80 \) feet → NO home run
Student Directions

Draft Directions: This is a list of eligible players. Each team will alternate picking players until all players are chosen. Circle the players that are drafted to your team.

Gincarlo Stanton
Gary Sanchez
Mike Moustakas
Miguel Sano
Aaron Judge
Justin Bour
Cody Bellinger
Charlie Blackmon

Derby Directions For each round choose a player to go up to bat. On your worksheet, find that player’s information. Plug in the information into the corresponding equations. Solve the equations to determine if the player hit a home run. Make sure to fill out all necessary information on your worksheet, including your work. You will turn the sheet in at the end of the activity.

Background For this activity you will need to use the information we’ve learned in this unit about parametric equations. You will apply your knowledge of parametric equations to model and solve problems in the context of baseball. Why parametric equations? We can use parametric equations to describe the motion of an object. Parametric equations can describe the motion of a ball as a function of time. We can specify where the object travels and how long it takes to get there. We will use parametric equations to model these situations.

Needed Information:
Distance to outfield wall- 407 ft.
Height of outfield wall- 8 ft.

\[ x = v_0 \cos \theta \cdot t \]
\[ y = h_0 + v_0 \sin \theta \cdot t - \frac{1}{2} \cdot 16 \cdot t^2 \]

where \( v_0 \) is initial velocity (feet/second)
\( \theta \) is angle of contact
\( t \) is time (seconds)
\( h_0 \) is initial height
Draft Prospects

**Gincarlo Stanton** hits the ball at 100 feet/second. He contacted the ball 3 feet above home plate at a 15° angle.

**Gary Sanchez** hits the ball at 125 feet/second. He contacted the ball 3 feet above home plate at a 45° angle.

**Mike Moustakas** hits the ball at 120 feet/second. He contacted the ball 2 feet above home plate at a 35° angle.

**Miguel Sano** hits the ball at 130 feet/second. He contacted the ball 4 feet above home plate at a 30° angle.

**Aaron Judge** hits the ball at 152 feet/second. He contacted the ball 3 feet above home plate at a 20° angle.

**Justin Bour** hits the ball at 115 feet/second. He contacted the ball 2 feet above home plate at a 45° angle.

**Cody Bellinger** hits the ball at 120 feet/second. He contacted the ball 3 feet above home plate at a 30° angle.

**Charlie Blackmon** hits the ball at 145 feet/second. He contacted the ball 3 feet above home plate at a 20° angle.
Student Worksheet

Round 1

Player: _______________________

\[ V_0 = \quad h_0 = \]

\[ \theta = \quad t = \]

Equations: ________________  ________________

Work:

Home Run? Yes OR No. How do you know?

Round 2

Player: _______________________

\[ V_0 = \quad h_0 = \]

\[ \theta = \quad t = \]

Equations: ________________  ________________

Work:

Home Run? Yes OR No. How do you know?
Round 3

Player: ______________________

\[ V_0 = \quad h_0 = \]

\[ \theta = \quad t = \]

Equations: __________________  __________________

Work:

Home Run? Yes OR No. How do you know?

Round 4

Player: ______________________

\[ V_0 = \quad h_0 = \]

\[ \theta = \quad t = \]

Equations: __________________  __________________

Work:

Home Run? Yes OR No. How do you know?
Legos: Addition and Subtraction of Fractions

Hailey Haas

Introduction: Legos are a great form of a manipulative to mix it up from the typical base ten block lessons. Students may play with Legos at home therefore will be able to associate fun with this lesson. It might be interesting to relate this lesson to construction of buildings and using materials to create a final “whole” product.

NCTM Standards: 4.NF.3.a – Understand addition and subtraction of fractions as joining and separating parts referring to the same whole

Objectives:

- Students will be able to form addition and subtraction equations using fractions.
- Students will be able to model this understanding through Legos as a representation of parts of a whole number.
- Students will be able to form fractions of the total number of blocks used by grouping of colors

Learning activities:

1. Students will create 4 addition/subtraction equations using Legos to represent a whole.
2. Students will convert the blocks into fractions and create number equations.
3. Students will count the number of blocks of each color used. With this the students will form fractions of the total number of blocks used grouped by color.

Formative assessment:

- Grading the student worksheet
  - Look for consistency relating to the whole, use of fractions, use of blocks, etc.
- Use the attached checklist while the students complete this task
Lego Math Checklist

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Using Legos as Instructed</th>
<th>Using both addition and subtraction</th>
<th>On task</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
Exploring Fractions with Legos

1. Take 18 Legos from the box
2. Choose a block to represent 1 (a whole)
3. Form 4 different equations using addition and subtraction to represent the whole or part of the whole

For example:

\[
\begin{align*}
\begin{array}{c}
\text{\includegraphics[width=0.2\textwidth]{image1.png}} \\
\text{\includegraphics[width=0.2\textwidth]{image2.png}} \\
\end{array}
\end{align*}
\]

= 1

4. Layout these equations with the blocks on a piece of paper. Draw the equations you made with blocks below:

1.

2.

3.

4.

5. Write the equations that you created with fractions in the space below

1.

2.

3.

4.
Exploring Fractions with Legos

6. Count the number of each color used. Represent this as a fraction of the total blocks used.

<table>
<thead>
<tr>
<th>Block Color</th>
<th>Fraction of Total Blocks (18)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
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</tr>
</tbody>
</table>
Teacher Guide – Exploring Fractions with Legos

1. Ensure that you have enough Legos for each student to have 18 blocks
2. Give clear instructions for students:
   a. If needed, give an example of how many ways you can represent the “whole” block – ex:

   ![Example of fractions with Legos]

   \[
   \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1
   \]
   \[
   \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}
   \]

3. Make sure that students know the difference between representing the equations with the blocks versus with fractions
4. Be sure to fill out the checklist while students complete this their worksheet
Fun with Fractals

Sarah Roberts

Introduction

Fractals are figures made up of an infinite repeated pattern such that this pattern is made up of smaller copies of the larger picture. They can be found on natural objects such as flowers, pinecones, and seashells, but they can also be used to model features of landscapes such as coastlines, forests, and trees. There are many fractals, but we will focus on the Sierpinski Triangle to discover patterns about the area, perimeter, and number of triangles contained in iterations of the triangle.

Grade/Course

Math 3

NCSCOS Standards

NC.M3.A-CED.1 : Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

NC.M3.A-CED.2 : Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Objective(s)

- Students will be able to draw iterations of the Sierpinski triangle.
- Students will be able to develop equations in terms of n that represent area, perimeter, and number of triangles in each iteration of the Sierpinski triangle.
Learning Activity:

- In our activity we explore the area, perimeter, and number of white triangles in the \(n^{\text{th}}\) iteration of the Sierpinski triangle. We begin by asking students to predict what will happen to the area, perimeter, and number of black triangles in the Sierpinski triangle. They will be asked to predict whether these values increase or decrease, and to write their predictions down with justification. Next they will construct the first four iterations of the Sierpinski triangle on a worksheet containing four blank triangles, and instructions on how to draw the first iteration of the Sierpinski triangle. This will help them understand the visual pattern, and lead them to be able to write equations describing the patterns they see in the area, perimeter, and number of triangles. Then we will have the students begin to calculate the area, perimeter, and number of triangles while looking for a pattern. Students will record their findings in a table provided for them. They will also record this data in graphs that will also be provided. We will spend the end of class cutting out our fourth iteration of Sierpinski’s triangle in order to calculate and build the largest Sierpinski triangle we can. See the attached teacher activity sheets and handouts.

Formative Assessment:

The student handouts will be collected at the end of class. It will be graded on accuracy, completion, and effort. Effort will be calculated by the amount of detail students include in their responses. Students will also be encouraged to discuss the worksheets with each other during class. The checklist on the third teacher handout will be filled out for each student as class goes on.
Student Handout 1

The Sierpinski triangle is drawn by connecting the three midpoints of the outline of a right triangle, this creates an upside triangle in the center of the outline of the right triangle we started with. Color the triangle you have just created black. This is then done to all white triangles created by drawing the black triangle. The first iteration (n=0) is simply the outline of a right triangle.

Fill in the following iterations of the Sierpinski triangle:

n=0
(complete)

n=1
Predictions:

What happens to the area of the black triangles in the Sierpinski triangle as the number of iterations approaches \( n \)?

What happens to the perimeter of the black triangles in the Sierpinski triangle as the number of iterations approaches \( n \)?

What happens to the number of the black triangles in the Sierpinski triangle as the number of iterations approaches \( n \)?
### Student Handout 2

Fill in the following tables for the first four iterations of the Sierpinski triangle.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Area of black triangles</th>
<th>Perimeter of black triangles</th>
<th>Number of black triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n=1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n=2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n=3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now that you have recorded your data, graph it to see a pattern.
While calculating the area, perimeter, and number of black triangles in each iteration of the Sierpinski triangle, what patterns did you notice. Please describe these below.

Can you predict area, perimeter, and number of black triangles for n=5? n=6?

Can you write area, perimeter, and number of black triangles in terms of n?
Teacher Handout 1

We create the Sierpinski triangle by connecting the three midpoints of an equilateral triangle, which makes four new equilateral triangles. Next we remove the middle triangle. This creates an upside triangle in the center of the outline of the right triangle we started with. We draw each iteration by repeating this process for all of the remaining black triangles. We begin with $n=0$, which is simply the outline of an equilateral triangle.

Fill in the following iterations of the Sierpinski triangle:

$n=0$
(complete)

$n=1$
n=2

n=3

Predictions:

What happens to the area of the black triangles in the Sierpinski triangle as the number of iterations approaches $n$?

The area approaches zero.

What happens to the perimeter of the black triangles in the Sierpinski triangle as the number of iterations approaches $n$?

The perimeter approaches infinity.

What happens to the number of the black triangles in the Sierpinski triangle as the number of iterations approaches $n$?

Approaches infinity, the general equation is $y=3^{n-1}$. 
### Teacher Handout 2

Fill in the following tables for the first four iterations of the Sierpinski triangle.

(For this part it is easiest to use two different triangles, and assume that the area for the n=0 iteration is 1, and assume that the side lengths of the n=0 triangle are 1.)

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Area of black triangles (Assume the area of the n=0 triangle is 1)</th>
<th>Perimeter of black triangles (Assume the perimeter of the n=0 triangle is 3)</th>
<th>Number of black triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=0</td>
<td>Each triangle area: A= 1</td>
<td>Each triangle perimeter: P= 3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total area: A=1</td>
<td>Total perimeter: P=3</td>
<td></td>
</tr>
<tr>
<td>n=1</td>
<td>Each triangle area: A=.25</td>
<td>Each triangle perimeter: P= 3/2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Total area: A=.75</td>
<td>Total perimeter: P=9/2</td>
<td></td>
</tr>
<tr>
<td>n=2</td>
<td>Each triangle area: A= 1/16</td>
<td>Each triangle perimeter: P= 3/4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Total area: A=9/16</td>
<td>Total perimeter: P=27/4</td>
<td></td>
</tr>
<tr>
<td>n=3</td>
<td>Each triangle area: A= 1/64</td>
<td>Each triangle perimeter: P=3/16</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Total area: A=27/64</td>
<td>Total perimeter: P=81/8</td>
<td></td>
</tr>
</tbody>
</table>
Now that you have recorded your data, graph it on the following page and see if you can find a pattern.
While calculating the area, perimeter, and number of black triangles in each iteration of the Sierpinski triangle, what patterns did you notice. Please describe these below.

\[ N = 3^n \text{ (Total number of triangles)} \]
\[ P = 3 \left( \frac{3}{2} \right)^n \text{ (Total perimeter of triangles)} \]
\[ A = \left( \frac{3}{4} \right)^n \text{ (Total area of triangles)} \]

Can you predict area, perimeter, and number of black triangles for \( n=5 \)? \( n=6 \)?

\[ N(5) = 3^5 \quad N(6) = 3^6 \]
\[ P(5) = 3 \left( \frac{3}{2} \right)^5 \quad P(6) = 3 \left( \frac{3}{2} \right)^6 \]
\[ A(5) = \left( \frac{3}{4} \right)^5 \quad A(6) = \left( \frac{3}{4} \right)^6 \]

Can you write area, perimeter, and number of black triangles in terms of \( n \)?

\[ N = 3^n \text{ (Total number of triangles)} \]
\[ P = 3 \left( \frac{3}{2} \right)^n \text{ (Total perimeter of triangles)} \]
\[ A = \left( \frac{3}{4} \right)^n \text{ (Total area of triangles)} \]
Teacher Handout 3

Checklist

1. Student is utilizing the specific examples to understand the general cases.
2. Student is properly using the formulas for area and perimeter.
3. Student is properly drawing the iterations of the Sierpinski triangle.