To facilitate student interest, we identify personal, cultural and community assets of a particular class context, and then use that information to plan instructional activities that connect with students’ assets. This work is in line with edTPA requirements for preservice teachers, and also helpful for inservice teachers.

Choreographing Fractions – Julianna Miller .........................................................2
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The Game of Life – Alex Morgan .................................................................11
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Questions or problems, please contact Dr. Leah McCoy mccoy@wfu.edu
In this project, students become familiar with adding fractions with like denominators through an arts integrated activity. Students vote on song to dance to and each student choreographs their own dance. Each dance consists of a total of 8 steps, where each step is either on a red, blue, yellow, or green spot. Students calculate the fractions of their total number of steps that are on the yellow spot, green spot, blue spot, and red spot. Then, students create and solve fraction addition problems that reflect the total number of steps on two colored spots.

Fourth Grade
• **Common Core Math Standards**
  • 4.NF.B.3 Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$.
  • 4.NF.B.3.D Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
• **Targeted Standard for Mathematical Practices:**
  • Make sense of problems and persevere in solving them.
  • Model with mathematics.
• **Objectives**
  • SWBAT create fractions from choreography steps
  • SWBAT create and solve fraction addition equations with like denominators using choreography steps

**Learning Activity**

Preparation: Students work in teams of two and each team is given four spots. These spots are arranged on the floor to make a square (one spot per corner of the square). Each spot is a different color: red, blue, green, and yellow. Students vote on the song (out of three choices pre-approved by the teacher) to which they will choreograph their dance.

Activity: Both students are told they have a total of 8 dance steps (this will be the denominator in our fractions). They must make each step on one of their four colored spots. Students take turns choreographing, while the other student tallies the number of dance moves are on each colored spot. Then, students add their total number of green, red, blue, and yellow moves.

Once students have finished their dances and have recorded their moves on their sheets, they complete the rest of the questions. These questions guide them through creating fractions out of their dance moves and creating fraction addition equations.

For Example:
The choreographer touches their foot for 3 out of the 8 dance moves on blue, 2 out of the 8 dance moves on yellow, 1 out of 8 on green, and 2 out of 8 on red. The recording student writes these fractions 3/8 for blue, 2/8 for yellow, 1/8 for green, and 2/8 for red. Students then create a fraction addition equation by asking: In my dance, what fraction of my moves were on the yellow and red spots? Their partner would then solve this equation by writing 2/8 + 2/8 = 4/8, or 1/2.

After solving their equations, students have the option to perform their dance for the rest of the class.

**Formative Assessment**

The teacher will visit each pair to watch their dances and observe the creation of the fraction equations. Students will each individually produce a copy of their fraction equations by turning in their recording sheet. The teacher will also obtain formative assessment data from the recording sheet to see if equations were accurately solved.
# Activity Sheet for Students

<table>
<thead>
<tr>
<th>Name:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner’s Name:</td>
<td></td>
</tr>
<tr>
<td>Date:</td>
<td></td>
</tr>
</tbody>
</table>

What is the **total number of steps** in your dance?

What fraction of steps in your dance are on the **blue spot**?

What fraction of steps in your dance are on the **yellow spot**?

What fraction of steps in your dance are on the **green spot**?

What fraction of steps in your dance are on the **red spot**?

Write the addition **equation** to represent the fraction of steps on the **yellow** or **blue** spot.

Write the addition **equation** to represent the fraction of steps on the **yellow** or **red** spot.

Write the addition **equation** to represent the fraction of steps on the **red** or **blue** spot.
Write the addition **equation** to represent the fraction of steps on the **green** or **blue** spot.

---

Write the addition **equation** to represent the fraction of steps on the **green** or **yellow** spot.

---

Write the addition **equation** to represent the fraction of steps on the **red** or **green** spot.
Students will learn about two- and three-dimensional shapes by integrating children’s literature into a mathematics lesson. The teacher will read, “Captain Invincible and the Space Shapes” by Stuart J. Murphy to the whole class. Students will work in groups to make their own control panel to their spaceship using three-dimensional shapes. They must discuss what each shape is, the vocabulary that goes with it and what its purpose will serve. Students will then trace the shapes and discuss how these three-dimensional shapes changed into two-dimensional shapes.

**Kindergarten - NCSCOS**

- **Math Standards**
  - NC.K.G.2 - Correctly name squares, circles, rectangles, hexagons, cubes, cones, cylinders, and spheres regardless of their orientations or overall size.
  - NC.K.G.3 - Identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres as two-dimensional or three-dimensional.
  - NC.K.G.4 - Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, attributes and other properties.
Objectives

- SWBAT correctly name shapes regardless of their orientation or size.
- SWBAT identify shapes as two- or three-dimensional.
- SWBAT talk about shapes using informal language to discuss similarities, differences, attributes and other properties.
- SWBAT analyze and compare two- and three-dimensional shapes.

Learning Activities

Activity One

The teacher will read the book, “Captain Invincible and the Space Shapes” to the whole class. The teacher can choose to read this book out loud themselves with or without projecting each page onto a promethean board. (There are videos of this book being read on youtube as well if one doesn’t want to purchase the actual book) During this time make sure to ask questions about the shapes in the book and the vocabulary students need to know (base, vertex etc.). A helpful strategy for teachers is to place sticky notes over some words so students won’t know the answer to a question you want to ask. For example, on page 10 it reads, “Comet, push the first button after the square!” The next line explains what shape that is, but if the teacher covers this up so the students can’t see it, this is a perfect time to ask the students what shape comes after the square.

During this time, the teacher can also pass around foam three-dimensional shapes as each shape is revealed throughout the book (depending upon school resources).

Activity Two

Students will then be placed into groups where they will be able to make their own control panels for their spaceship. Groups should be of three to four students. Students will be using three-dimensional shapes that they see in the environment around them tissue box, traffic cone, party hat etc.). They will discuss what each shape is, its attributes and its function on the control panel. Once students have completed their control panels, they will trace their three-dimensional shapes. Students will then discuss how their three-dimensional shapes turned into two-dimensional shapes. In the end, each group will present their control panel to the whole class.
Formative Assessment

The teacher will use the student’s control panels, group presentations, field notes and observation as a formative assessment. As students are working in their groups the teacher will need to manage their time to observe and work with each group. When doing this the teacher will need to take notes on what she is seeing (or shortly after lesson).
Activity Sheet for Teacher Use
Captain Invincible and the Space Shapes Activity

Learning Activity:

- Teacher reads book aloud to whole class - with or without projecting the book onto a promethean board (using the link below, one can also find this book on youtube)
- Teacher frequently stops to ask questions (ex. What is this part of a cone called?, How is the square different from the other shapes in the row on the control panel?)
- Teacher passes around three-dimensional shapes to class as they are revealed throughout the book
- Students work in groups on control panels (see picture below)
- Students discuss the three-dimensional shapes provided for them and their use on the control panel
- Students trace three-dimensional shapes and discuss how they become two-dimensional shapes (see picture below)
- Each group will present their control panels to the whole class

Resources:

Book: Captain Invincible and the Space Shapes, By: Stuart J. Murphy

Read Aloud on Youtube: https://www.youtube.com/watch?v=NIGk-CZhdIM

Foam Three Dimensional Shapes

Common Objects That Are Three Dimensional Shapes
Student Example: (author)
The Game of Life - Counting Money and Exchanging Bills!

Alex Morgan

Introduction:

In this activity, second graders will go through a life-sized board game that will have them working with dice, fake currency manipulatives, and game board pieces that require reading comprehension to understand why they are gaining fake money throughout the game. They will be asked to use their knowledge of place value to eventually exchange 10 single $1 bills for $10 bills at the “bank.” Students will go through the game as a pair in order to maintain timeliness in the class. They will get only $1 bills throughout the game. Once the pair gets 2 turns, they will
count the money they have gotten and be asked to identify which number is in the ten’s place. They will have to go to the “bank” which is the teacher who has all of the $10 bills. They will be asked to bundle the money into groups of 10 singles in each pile, plus a pile for the money that does not fit into a group of 10. The bank will then substitute 10 singles for a $10 bill. After the students are done at the bank, they will confer with other student pairs about how much money they have at the end of the game. Some student pairs will be asked to present their conclusion to the class about which pair had more money and why.

Second Grade-NCSCOS

NC.2.NBT.6- Add up to three two-digit numbers using strategies based on place value and properties of operations.

Objectives

- SWBAT count how many spaces to go in a board game based on the number they roll on a die
- SWBAT work with a peer to navigate the game and read the “life” result on the space they have landed on and count the amount of money they are receiving in fake currency which is dependent on the space on which they land.
- SWBAT calculate how much money they have at the end of the game
- SWBAT exchange ten single dollars for a ten-dollar bill
- SWBAT confer with other student pairs about how much money they have at the end of the game
- SWBAT to calculate which student pair has more money between the 2 pairs

Learning Activities

1. Students will work in pairs to navigate a life-sized board game- “the game of life” by rolling a die and landing on spaces that equate to specific scenarios that result in a gain fake currency money. Student pairs will each roll the die two times and then the game will be over. The board game can be created by the teacher using construction paper, markers, fake currency, and washi tape.
2. On each space, students will win fake money—all of which will be in single dollar bills.
3. At the end of the game, the pair of students will have to calculate how much money they have. They will have to then exchange bundles of ten single dollar bills for a ten-dollar bill at the “bank” (the teacher).
You found a lemonade stand and earned $6.
You did your chores and got a $5 allowance.
You took all the lemons off your tree and earned $15.
You sold lemonade in a bottle case for $4.
You went shopping for a toy and got $10.
You returned a lost item and got $10.
You wanted the lemonade stand for a week. You got $8.
Good work! You get your $7 allowance.
If you wore your birthday hat, you got $6 from your grandparents.
You won a prize with a gift price.
Formative assessment: think-pair-share

- Each time a student pair goes to the bank to exchange ten single dollar bills for one $10 bill, the teacher will count with them to ensure procedural fluency and understanding of place value.
- At the end of the game once all students have exchanged their singles for $10 bills, each pair of students will find another pair and count each other’s money. They will determine which student pair had more money at the end of the game and won the “game of life.”
- All student pairs will be asked to share if they won or lost the game of life and how they know based on their standing with the other team with whom they conferred.

Student example of mastery:

A student pair rolls a 4 on the first round and gets $11. They roll a 5 on the next round and win $14. They use the fake currency manipulatives to count out their winnings of $25. They then recognize that there is a 2 in the ten’s place—meaning $20 of their single dollar bills can be replaced with 2 ten-dollar bills. They will come to the bank and give 20 single dollars to the banker (the teacher) which they will all count together to ensure proficiency. They then have 2 $10-dollar bills and 5 $1 bills. After they are done at the bank, they find another pair of students who has 1 $10 bill and 7 single dollar bills. They conclude that even though Pair 2 has more physical bills, Pair 1 has won the game of life because they have more money. They present their findings to the class, and they have exhibited conceptual understanding of the topic.
Student example of proficiency:

A student pair rolls a 1 on the first round and gets $15. They roll a 6 on the next round and win $8. They use the fake currency manipulatives to count out their winnings of $23. They go to the bank and ask for 2 $10 bills. Instead of handing the teacher at the bank $20 in singles, they hand all of their money over. With the teacher’s help, they count out $23 in 2 $10 bills and 3 single $1 bills. After they are done at the bank, they find another pair of students who has 1 $10 bill and $7 single dollar bills. They conclude that Pair 1 has more of the currency manipulatives, therefore, they have more money. They present their findings to the class. The class is given an opportunity to critique the works of Pairs 1 and 2. After the teacher facilitates a discussion about why their conclusion is wrong, Pair 1 is able to explain back to the class the actual correct answer for who would have more money. Pair 1 has exhibited proficiency but needs some teacher and outside assistance to understand all of the points of the activity.
Student example of nonfulfillment:

A student pair rolls a 5 on the first round and gets $10. They roll a 2 on the next round and win $8. They use the fake currency manipulatives to count out their winnings of $18. They notice that most of the students around them have numbers in the 20s. They then conclude that due to this, they do not have to go to the bank to exchange any money because they have less money than their peers at the end of the board game. They have $25 total. They conclude Pair 2 has won the game because they have more money at the end of the game. They present their findings to the class, and even though they have correctly concluded that Pair 2 has more money, both pairs failed to recognize that Pair 1 did not have their money in the simplest form with having exchanged 10 singles $1 for a $10. These students have displayed that the class may benefit from the teacher doing an example and guiding the class through it.
Introduction: This lesson investigates shapes such as scalene triangles and quadrilaterals, and how to identify their congruent counterparts. This lesson utilizes origami, which is the art of paper folding that originates from Japan. Using origami, you can transform a flat sheet of square paper into different shapes. In this particular lesson, students will learn how to make a whale out of origami. Folding origami into a whale consists of many triangles where students will be able to identify congruent triangles as well as identifying the number of triangles by realizing that there are smaller triangles in bigger triangles. The final result of the whale will be a quadrilateral, where students can explore whether those sides match.

Grade/Course: 4th Grade NC.4.G.2 - Classify quadrilaterals and triangles based on angle measure, side lengths, and the presence or absence of parallel or perpendicular lines.

Objective: As a result of this lesson, students will be able to identify congruent triangles and scalene triangles.

Learning Activity:
1) Teacher will start off by reviewing important math vocabulary that will be necessary for this activity
   - Quadrilateral: Any figure that has four sides
   - Square: A quadrilateral that has four right angles (90 degrees) and four congruent sides
   - Line of Symmetry: A line that divides two halves that match
   - Congruent: Equal in measurement
   - Triangle: Any figure that has three sides
   - Scalene Triangle: A triangle that has no sides that are the same length
2) Teacher will hand out origami directions and origami paper to the class. Together as a class, with the teacher showing on an overhead projector, everyone will make the whale using origami. Integrate the above vocabulary when make the whale.
   ● The shape you start off with is a **square**. If you turn the square to its point you make a diamond
   ● In step 1, when you fold the diamond in half, you have created a **line of symmetry** down the center
   ● In step 2, when you fold the diamond in half, you have made **congruent** figures
   ● In step 2, these new triangles are **scalene triangles**

3) Pair share, and then explore as a class. Have students pair up and explore these questions. Then, regroup as a class and have people share their answers and have them justify their answers.
   ● After completing step 3, how many triangles can you count? (Remember there are smaller triangles in larger triangles)
   ● Find all the congruent triangles. How many pairs are there?
   ● In step 5, you have made a quadrilateral. Do any of these sides match?

**Formative Assessment:** Hand out exit ticket and have students identify which triangles are congruent and scalene.
Teacher’s Resources

Directions

Step 1  Start with a six-inch square, positioned like a diamond. Fold the left point over to meet the right. Open it up again.

Step 2  Fold the two sides inward to meet the center fold, or line of symmetry.

Step 3  Fold the top point down to meet the folded triangles.

Step 4  Fold the right side over to meet the left side.

Step 5  Rotate the shape so that the long, flat line is at the bottom.
Formative Assessment

Which triangles are congruent?
Which triangles are scalene?

1  2  3  4  5  6  7  8  9  10  11  12

Answers:

Which triangles are congruent? (b&f)

Which triangles are scalene? (2, 6, 4, 8, 10, 12)

Resources:
Math is Sweet: Cookies as Context

Hannah Maness

Introduction: Multiplication, division, and customary measurement are mathematical concepts that are crucial for students to master, not only for the test at the end of the year, but more importantly for real life. For students to master these concepts, they must be engaged in the way in which they are presented. Teachers should take advantage of the numerous real-life applications these concepts have to offer in order to increase engagement and give the content deeper purpose. One context to use to teach these concepts is baking. The activities described below, which are all about cookies, show just how sweet math can be.

Grade: Third

NCSCOS Standards:
- NC.3.OA.1, NC.3.OA.2, NC.3.OA.3 - Represent and solve problems involving multiplication and division.
- NC.3.MD.2 - Solve problems involving customary measurement.

Objectives: By the end of the activities, students will be able to (SWBAT)…
- Use and explain different strategies for solving multiplication and division problems.
- Interpret factors as representing equal groups and the number of objects in each group.
- Interpret the divisor and quotient in a division equation.
- Represent, interpret, and solve one-step multiplication and division word problems.
- Solve problems that involve customary measurement.
- Estimate and measure capacity and weight in customary units to a whole number.
Learning Activities:

Activity One:
- The teacher will read aloud the book *The Doorbell Rang* by Pat Hutchins. The first time the book is read, it should be during read aloud time just for fun.
- The second time the book is read, the teacher should give each student a copy of the table provided below. Additionally, each student should get 12 small cookies to use as manipulatives. (Tip: Use Cookie Crisp cereal.) The teacher should stop before each time the number of cookies per person is revealed in the book, so that students can manipulate their cookies, fill out their tables, and figure out the division problem for themselves.
- Students should first write how many cookies there are in total in the first column. This is always 12. In the second column, they should write how many people there are total, which increases in the book each time the doorbell rings. Next, they should move their cookies around to represent the problem. In the third column, students should draw a picture to represent the division that is happening each time the doorbell rings. This should connect to the grouping they created with their cookies. Finally, in the fourth column, students should write the answer for how many cookies each person should receive.
- Once all four columns have been filled out, the teacher can reveal the answer by continuing to read the book aloud.
- As students fill in their tables, the teacher should monitor the room and select students to demonstrate how they arrived at their answers, highlighting a variety of strategies.
- The selected students should briefly explain to their peers how they got their answers while the teacher reads the book aloud for the third time. During this final read aloud, students may also eat their cookies!

Activity Two:
- Students should each receive a copy of the word problems, which are all about cookies and baking. The word problems encourage students to make sense of customary units of measurement such as cups and pounds.
- Students should complete the word problems using a variety of strategies, as they prefer.
- Once they have completed the problems individually, the teacher may pair them up to discuss their answers.
- Finally, the teacher should lead the whole class in a discussion about the word problems, providing clarification and further explanation as needed.

Activity Three:
- Students will estimate and measure the amount of various ingredients that are necessary to make chocolate chip cookies.
- The teacher should prepare the ingredients in see-through containers beforehand, placing the following amount of each ingredient in separate containers.
  - Chocolate chips: 1 cup
  - Flour: 1 cup
  - White sugar: 2 cups
  - Brown sugar: 3 cups
  - Milk: 1 quart
• Additionally, the teacher should have unopened packages of the following ingredients, which are typically found in the grocery store in the amounts given.
  o Bag of chocolate chips: 12 ounces
  o Bag of flour: 5 pounds
  o Bag of white sugar: 4 pounds
  o Bag of brown sugar: 2 pounds
• The measurement labels on the unopened packages should be covered for the activity.
• Additional materials needed are: paper towels, 4 one-cup measuring cups, 4 empty containers for students to pour measured ingredients in, and 4 scales.
• Note: The milk can be kept in a quart jar, then students will not have to pour it into another container.
• Students should be grouped in pairs or trios, and each student should receive a copy of the recording sheet, which provides a table and additional questions for reflection.
• The ingredients should be placed around the room (at 9 different stations), and students should rotate to each station. First, they should record their estimate for the ingredient at each station. Then, they should work together to measure the ingredients as carefully as possible, placing them in the empty containers as they empty the measuring cup. Once they have the actual measurements, they should record them on their sheets.
• For the unopened, bagged ingredients, if scales are not available, the teacher can reveal the weight at the end of the activity.
• After all students have been to every station, they should be given a few minutes to talk in their groups about the reflection questions. Then, they should complete the questions individually after returning to their seats.

**Formative Assessment:**

• The teacher should monitor students while they complete the *The Doorbell Rang* table.
  o Tip: Write student names on a sticky note, in order of how they should present their findings during the third read of the book. Ask students to make sure that they are okay with sharing in front of their peers.
• The teacher should monitor students while they complete the multiplication and division word problems, looking for conceptual understanding and procedural fluency. A checklist with student names and space to write can be helpful for this. Additionally, the teacher should look for points to bring up in the whole class discussion at the end of the activity.
• The teacher should monitor students as they estimate and measure ingredients. Specifically, she/he should focus on reasoning and sense making in student discourse, and offer scaffolding as needed. The reflection questions at the end of the activity may be used to check student understanding.
<table>
<thead>
<tr>
<th>How many total cookies?</th>
<th>How many people?</th>
<th>Draw it out!</th>
<th>How many cookies should each person get?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Phoebe has 20 chocolate chip cookies to share among herself and 9 other people. How many cookies should each person get?

2. Miguel wants to give each of his friends 3 cookies at lunch. If he has 9 friends, how many cookies does he need in all?

3. Jess and Cece need to bake 4 batches of cookies for their party. If 2 eggs are needed for each batch of cookies, how many eggs do they need altogether?

4. Winston used 16 cups of chocolate chips while baking cookies. If he used 2 cups of chocolate chips in each batch, how many batches of cookies did he make?

5. Kate bought 40 pounds of sugar for her bakery. If each bag of sugar weighs 4 pounds, how many bags of sugar did she buy?
<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Estimate</th>
<th>Actual Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate chips (in cups)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flour (in cups)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>White sugar (in cups)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brown sugar (in cups)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milk (in cups)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bag of chocolate chips (in ounces)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bag of flour (in pounds)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bag of white sugar (in pounds)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bag of brown sugar (in pounds)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Observations and Reflections:**

1. What surprised you, if anything?

2. Does one cup of chocolate chips look the same as one cup of flour? Why or why not?

3. How many cups of milk were there? How about pints? How about quarts? What does this tell you about the relationship between cups, pints, and quarts?

4. If you were baking a batch of chocolate chip cookies, what unit of measurement do you think would be used for the flour in the recipe?
Olympic Swimming – Does Height Matter?

Melissa McGahan

Introduction: In this lesson students will begin by exploring the circular pattern that a swimmers’ arms make when swimming backstroke. The students will then think of the length of one’s arm as the radius of a circle with the shoulder being the circle center. Students will collect data of arm length and compute the circumference of the circle their arm would make during backstroke. They will then graph the linear relationship between circumference and radius. Additionally, the students will be analyzing Olympic data of 100 meter backstroke swimmers that won a Gold medal in the Olympics. The students will graph the data points for each swimmer provided and examine the relationship between the two quantities. Students will consider whether a swimmer’s height is correlated to the swimmer’s time of swimming 100 meters. In general, the Olympics is a well-known event that has athletes from a wide variety of countries that students could relate to. Similarly, the lesson focuses on swimming, which students may understand.

NCSCOS:
- **NC.M1.A-CED.2**: Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.
- **NC.M1.S-ID.6**: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Objectives:
- SWBAT produce graphs of linear relationships between two variables.
- SWBAT analyze a graph of two quantitative variables and explain the relationship between the variables.
Learning Activities:

Activity 1 – *A Look at How a Swimmers’ Arms Relate to Circles*

- During this activity the teacher will briefly explain that a backstroke swimmer rotates their arms 360 degrees in a circular pattern. The teacher will then divide the students in the class into groups of 4 each and give each group a yard stick. The students will measure the length of one of their arms (See Resources section) and record the data for their whole group. Then the students will be using the circumference of a circle formula and calculate the circumference of the circle that their arm would make in the water during backstroke.

Activity 2 – *Does Height affect Speed of Olympic Gold Medalists?*

- During this activity the teacher will provide the students with a table of values for a swimmer’s height and the swimmer’s time to complete the 100 meter race in backstroke. The students will then graph the data points. Once the students graph the data points they will state a conclusion about the relationship between height of a swimmer and time to complete the 100 meters backstroke.

Formative Assessment:

For Activity 1 the students will turn in their worksheet from class depicting the graph of circumference and radius. The worksheet will be graded on correctness and the students will be asked to show their work so that the teacher can see what the student’s thought process was during the activity. The teacher should address any common mistakes that the class has during the subsequent lesson.

For Activity 2 the students will complete an Exit Ticket (See Resources section). The Exit Ticket asks the students to describe the relationship between a swimmer’s time to complete the 100 meters backstroke and their weight. This will be a way to assess each student’s ability to examine a scatterplot and explain the relationship between two quantities.
Resources:

Activity 1 (Student Version):

Swimming Backstroke – A Look at How a Swimmers’ Arms Relate to Circles

When performing backstroke the swimmer rotates each arm 360° continually.

Directions:

Using a yardstick, group members will measure the lengths of each other’s arms (in inches), from the shoulder to the longest fingertip. This length will be denoted as r (radius). Once each group is finished measuring the length of their arms, they will then calculate the circumference using the following formula: $C = 2\pi r$ (Note: use 3.14 for $\pi$)

<table>
<thead>
<tr>
<th>Name</th>
<th>Length of Arm (inches)</th>
<th>Circumference (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group Members</td>
<td>r =</td>
<td>C =</td>
</tr>
<tr>
<td></td>
<td>r =</td>
<td>C =</td>
</tr>
<tr>
<td></td>
<td>r =</td>
<td>C =</td>
</tr>
<tr>
<td></td>
<td>r =</td>
<td>C =</td>
</tr>
</tbody>
</table>

Directions:

Using the data from the table above, plot the (x,y) coordinate pairs, where x represents arm length and y represents circumference. Label the x-axis, y-axis and create a title for the graph.
Activity 2 (Student Version):

Swimming Backstroke – Does Height affect Speed of Olympic Gold Medalists?

Directions:

Below is a table of some of the Gold Medal Olympic Backstroke 100m swimmers. Begin by converting the height of each swimmer from feet into inches.

<table>
<thead>
<tr>
<th>Swimmer Name</th>
<th>Time (seconds)</th>
<th>Height (feet)</th>
<th>Height (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Males</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roland Matthes</td>
<td>56.58</td>
<td>6 feet 2 in</td>
<td></td>
</tr>
<tr>
<td>Aaron Peirsol</td>
<td>52.54</td>
<td>6 feet 3 in</td>
<td></td>
</tr>
<tr>
<td>David Theile</td>
<td>61.09</td>
<td>6 feet 3 in</td>
<td></td>
</tr>
<tr>
<td>Matthew Grevers</td>
<td>52.16</td>
<td>6 feet 8 in</td>
<td></td>
</tr>
<tr>
<td>Jeffrey Rouse</td>
<td>54.10</td>
<td>6 feet 4 in</td>
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</tr>
<tr>
<td><strong>Females</strong></td>
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<td></td>
</tr>
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<td>58.96</td>
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<td>Diana Mocanu</td>
<td>60.21</td>
<td>5 feet 7 in</td>
<td></td>
</tr>
<tr>
<td>Cathy Ferguson</td>
<td>67.07</td>
<td>5 feet 8 in</td>
<td></td>
</tr>
</tbody>
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(Swim times from: [https://www.olympic.org/](https://www.olympic.org/))

Directions:

Using the data from the swimmer table above, plot the (x,y) coordinate pairs, where x represents time (seconds) and y represents height (inches). Label the x-axis, y-axis and create a title for the graph.

Using the graph explain the relationship between time and height in the context of the problem:
Exit Ticket (Student Version):

Directions: Label the x-axis, y-axis, plot the data points and create a title for the graph.

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<thead>
<tr>
<th>Swimmer Name</th>
<th>Time (seconds)</th>
<th>Weight (lbs)</th>
</tr>
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<td></td>
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<tr>
<td>Roland Matthes</td>
<td>56.58</td>
<td>163</td>
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<td>52.54</td>
<td>201</td>
</tr>
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<td>Matthew Grevers</td>
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<td>229</td>
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</tr>
<tr>
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(Swim times from: [https://www.olympic.org/](https://www.olympic.org/))

Using the graph explain the relationship between time and weight in the context of the problem:

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________
Activity 1 (Teacher Version):

Swimming Backstroke – A Look at How a Swimmers’ Arms Relate to Circles

When performing backstroke the swimmer rotates each arm 360° continually.

Directions:

Using a yardstick, group members will measure the lengths of each other’s’ arms (in inches), from the shoulder to the longest fingertip. This length will be denoted as \( r \) (radius). Once each group is finished measuring the length of their arms, they will then calculate the circumference using the following formula: \( C = 2\pi r \) (Note: use 3.14 for \( \pi \)) (Note: Sample data below – data will vary)

<table>
<thead>
<tr>
<th>Name</th>
<th>Length of Arm (inches)</th>
<th>Circumference (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melissa</td>
<td>( r = 24.5 )</td>
<td>( C = 2 \times 3.14 \times 24.5 = 153.86 )</td>
</tr>
<tr>
<td>Dave</td>
<td>( r = 26.5 )</td>
<td>( C = 2 \times 3.14 \times 26 = 163.28 )</td>
</tr>
<tr>
<td>Rachel</td>
<td>( r = 25.25 )</td>
<td>( C = 2 \times 3.14 \times 25.25 = 158.57 )</td>
</tr>
<tr>
<td>Chris</td>
<td>( r = 24.75 )</td>
<td>( C = 2 \times 3.14 \times 24.75 = 155.43 )</td>
</tr>
<tr>
<td>Sarah</td>
<td>( r = 27 )</td>
<td>( C = 2 \times 3.14 \times 27 = 169.56 )</td>
</tr>
</tbody>
</table>

Directions:

Using the data from the table above, plot the \((x,y)\) coordinate pairs, where \( x \) represents arm length and \( y \) represents circumference. Label the x-axis, y-axis and create a title for the graph.
Activity 2 (Teacher Version):

Swimming Backstroke – Does Height affect Speed of Olympic Gold Medalists?

Directions:

Below is a table of some of the Gold Medal Olympic Backstroke 100m swimmers. Begin by converting the height of each swimmer from feet into inches.

<table>
<thead>
<tr>
<th>Swimmer Name</th>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Roland Matthes</td>
<td>56.58</td>
<td>6 feet 2 in</td>
<td>$12 \times 6 + 2 = 74$</td>
</tr>
<tr>
<td>Aaron Peirsol</td>
<td>52.54</td>
<td>6 feet 3 in</td>
<td>$12 \times 6 + 3 = 75$</td>
</tr>
<tr>
<td>David Theile</td>
<td>61.09</td>
<td>6 feet 3 in</td>
<td>$12 \times 6 + 3 = 75$</td>
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<tr>
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<td>52.16</td>
<td>6 feet 8 in</td>
<td>$12 \times 6 + 8 = 80$</td>
</tr>
<tr>
<td>Jeffrey Rouse</td>
<td>54.10</td>
<td>6 feet 4 in</td>
<td>$12 \times 6 + 4 = 76$</td>
</tr>
<tr>
<td><strong>Females</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Natalie Coughlin</td>
<td>58.96</td>
<td>5 feet 8 in</td>
<td>$12 \times 5 + 8 = 68$</td>
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<td>Krisztina Egerszegi</td>
<td>60.68</td>
<td>5 feet 9 in</td>
<td>$12 \times 5 + 9 = 69$</td>
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(Swim times from: https://www.olympic.org/)

Directions:

Using the data from the swimmer table above, plot the (x,y) coordinate pairs, where x represents time (seconds) and y represents height (inches). Label the x-axis, y-axis and create a title for the graph.

Using the graph explain the relationship between time and height in the context of the problem:
**Exit Ticket (Teacher Version):**

Directions: Label the x-axis, y-axis, plot the data points and create a title for the graph.

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Using the graph explain the relationship between time and weight in the context of the problem:

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________
Quadratics in the Odyssey

Daniel Spieler

Introduction: This lesson will have students take a voyage through the ancient Mediterranean as Odysseus did in the Odyssey! Students will visit the various locations from the epic in groups and solve a problem that Odysseus might have come across in his journey. These choices will be made by solving a quadratic equation or a system of equations that includes a linear and quadratic element. The students will interpret these solutions in this context to get a full picture of the scenario.
Course: Math II

NCSCOS Standards:
NC.M2.A-REI.1- Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.
NC.M2.A-REI.7- Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.

Objectives:
- SWBAT solve quadratic equations and determine what the roots mean and how they can be applied to a scenario
- SWBAT solve a system of equations with a linear and quadratic element to determine where they meet and then interpret that solution

Learning Activities:
- Odysseus’ journey worksheet (One paper per group)
- Class discussion/review of the problems that will ask each group to present an answer/part of an answer to a problem to the class on the board. The students will then agree/disagree with the answer and we will discuss the different ways of doing the problems that came up. Teacher should purposefully decide who will present as the activity is being done, noticing any errors/common misconceptions that arise that would be worthy of class discussion.
- “exit ticket”- Graph the equations $f(x) = (x + 1)^2$ and $f(x) = 4x + 4$ to see where they meet. Once you have done this, set them equal to each other and solve to check your answer.

Formative Assessment:
- Student groups will turn in their worksheet and the worksheet will be graded for correct methods and thinking processes. Teacher should also note thought processes shown in presentation to class to help with evaluating understanding.
- Students will be given a simple linear and quadratic equation to graph and then factor as an “exit ticket”. They must then check that their graph matches their answer from factoring. This activity serves to check that all students understood the activity and are able to do the work that was done in the groups on their own.
Decision-Making FACTORS in the Odyssey

Directions: In groups of 3 you will solve the problems below that Odysseus would have faced in his journey home from the Trojan War. For each problem, read the scenario and answer the questions. Once you have shown me your work, I’ll give you an explanation of what actually happened and the next problem. See how far you can get before we meet as a class and discuss the activity!

Stop 1- The Land of the Cicones:
Odysseus and his crew made their first stop after Troy at Ismarus. They then did what any self-respecting Greeks would do and looted and pillaged the town. While looting, the townspeople went to get reinforcements to drive Odysseus away. Odysseus and his men looted the town at a rate of \( f(x) = -x^2 + 10x + 6 \), where \( f(x) \) is number of gold bars and \( x \) is time in hours.
1. Decide when would be the best time for Odysseus to leave for them to have the most gold and how many gold bars would they have?
2. When would be the worst time for them to leave?
3. What method did you use for solving this problem and why?
4. In 2-4 sentences, describe what you think happened based on the equation.

Actual story: They looted and pillaged very quickly and successfully at first as was their plan. At this point, Odysseus thought it would be a good idea for them to leave, but his men decided that they liked gold more, so they stayed. They stayed too long and the Cicones (inhabitants of Ismarus) drove them out and killed 6 of their men. The lesson here is to only loot quickly or else you might get caught.
Stop 2- The Island of the Cyclopes:

After Odysseus and his crew left the Cicones they went to the island of the Lotus-Eaters, but that’s a boring story so we’re going to skip it. They then went to island of the Cyclopes where they again looted and pillaged, but this time they looted the cave of the Cyclops Polyphemus who was the son of Poseidon (not a good idea). He then trapped and ate a lot of the men until they were able to trick him and leave with him hot on their tails. Once on their ship they paddled away from shore at a rate of $f(x) = 20x$, where $f(x)$ is feet and $x$ is seconds. Polyphemus, standing at the shoreline, threw a boulder straight at them at rate of $f(x) = 18(x-5)^2$.

1. Draw the two equations on the same graph on graph paper. Approximately how many seconds would it take for the boulder to hit the ship? (Give a guess here)
2. Find out when the boulder would hit the ship and describe every step you took to find this out. A graphing calculator may be used, but only for arithmetic and complete work should be shown.
3. Describe what happened based on the two equations (be specific) in 2-3 sentences.

Actual Story: He actually threw two rocks and they both missed. But, the cyclops did tell his dad Poseidon and cursed Odysseus, causing him strife for the next 10 years. Odysseus learned here to never mess with people who have a Greek God for a dad, which apparently wasn’t common sense.
Stop 3- Aeolia and the Bag of Winds:

After leaving Polyphemus, the men traveled to Aeolia where Aeolus (the God of Winds) lives. He handed Odysseus a bag that contained all of the winds that would keep him from home so he could get there as fast as possible. Somehow this was a thing and after staying there for a day, they headed home at rate of $f(x) = -10x^2 + 200x -190$ where $f(x)$ is distance from Aeolia in miles and $x$ is time in days.

1. What does the negative sign in front of the quadratic element indicate about their journey?
2. If Ithaca, their home, is 850 miles away, did they reach it? Explain how you came to this conclusion.
3. Solve the equation, and along with the information from question 1 and 2, interpret what the roots mean and describe what might have happened to the crew.

Actual Story: Odysseus and his men sailed home after Aeolus on a journey that should have taken 10 days. Odysseus, not trusting his men to be strong-willed enough to not open the bag out of curiosity, didn’t tell them what was in it and somehow stayed awake for 9 nights. On the tenth, he awoke to find out that, sure enough, his men had opened the bag and the bad winds were pushing them on their way back to Aeolus after being so close to Ithaca. This, like most of the bad things that happened in the Odyssey, probably could have been avoided. A simple, “Don’t open the bag or we won’t get home” probably would’ve worked
Stop 4/5- Telepylos the land of the Laestrygonians/Aeaea the Island of Circe:
The next stop for the men is Telepylos, the land of the Laestrygonians. They are cannibals who eat people at a rate of \( f(x) = 7x + 5 \) where \( f(x) \) is people and \( x \) is time in minutes. The second is Aeaea the island of Circe, a Witch-God who turns men into pigs at a rate of \( f(x) = x^2 + 3x \). As I’m sure you can tell, neither of these visits went very well for the men.

1. Draw both rates on graph paper. Using this, decide at what point Circe turns more men into pigs than the Laestrygonians ate.
2. Write a table for the two rates, then decide who was more destructive to Odysseus and his crew if they only stayed at each place for 3 minutes, then do the same for 6 minutes. Explain why this is the case and why this makes sense.
3. Find the time where the number of people Circe turns into pigs equals the number of men the Laestrygonians ate using only the two equations. Explain why you used the method you used.

Actual Story: The Laestrygonians ended up eating a lot of people and even smashed 11 of Odysseus’ ships. They were bad news and the men spent very little time on their island. Circe, however, ended up being nice and feeding the whole crew (after she turned them all into pigs and Odysseus came to the rescue). They stayed on Aeaea for a year to rest up, as you can imagine, getting eaten twice and pillaging is pretty tiring.
Stop 1- The Land of the Cicones:
Odysseus and his crew made their first stop after Troy at Ismarus. They then did what any self-respecting Greeks would do and looted and pillaged the town. While looting, the townspeople went to get reinforcements to drive Odysseus away. Odysseus and his men looted the town at a rate of \( f(x) = -x^2 + 10x + 6 \), where \( f(x) \) is number of gold bars and \( x \) is time in hours.

1. Decide when would be the best time for Odysseus to leave for them to have the most gold and how many gold bars would they have?
2. When would be the worst time for them to leave?
3. What method did you use for solving this problem and why?
4. In 2-4 sentences, describe what you think happened based on the equation.

1. Students should solve using completing the square so that vertex can be found. Students should get \((x-5)^2 = 31\) and then decide that the best time would be after 5 hours and they would have had 31 gold bars.
2. Students should think about finding the root here as this would leave them with zero gold bars. They should then use the form found in the previous step and finish solving. They should get that the worst time would be at \(5 + \sqrt{31}\) hours or about 10.5 hours.
3. This question assesses their knowledge of the different information you can find when you complete the square versus when you use the quadratic equation. It also asks them to reflect on their thought processes while problem-solving.
4. Students should see that the gold increased until 5 hours and then dropped. This should indicate to them that the soldiers were fetched and they were driven out of the land and that the gold that was looted was taken back.

Actual story: They looted and pillaged very quickly and successfully at first as was their plan. At one point, Odysseus thought it would be a good idea for them to leave, but his men decided that they liked gold more, so they stayed. They stayed too long and the Cicones (inhabitants of Ismarus) drove them out and killed 6 of their men. The lesson here is to only loot quickly or else you might get caught.
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1. Draw the two equations on the same graph on graph paper. Approximately how may seconds would it take for the boulder to hit the ship? (Give a guess here)
2. Find out when the boulder would hit the ship and describe every step you took to find this out. A graphing calculator may be used, but only for arithmetic and complete work should be shown.
3. Describe what happened based on the two equations (be specific) in 2-3 sentences.

![Graph](image)

1. Students graph should look similar to the one above but will be less accurate as it will be hand-drawn. Students should hopefully not draw any values for the parabola before \( x = 5 \). A good guess here, would be around 8 seconds.
2. This task should be simplified with a calculator so that emphasis is not put on tiresome arithmetic. Students should explain their answers and whether they completed the square or did the quadratic equation. Students should also discuss which answer from solving the quadratic that they chose and why one works, and the other does not. If students have not discussed this on their paper, question them to make sure they know why.
3. Students should show their understanding that the rock was thrown after 5 seconds. This is testing whether the students understand what is happening based off of the graph and equations.

**Actual Story:** He actually threw two rocks and they both missed. But, the cyclops did tell his dad Poseidon and cursed Odysseus, causing him strife for the next 10 years. Odysseus learned here to never mess with people who have a Greek God for a dad, which apparently wasn’t common sense.
Stop 3- Aeolia and the Bag of Winds:

After leaving Polyphemus, the men traveled to Aeolia where Aeolus (the God of Winds) lives. He handed Odysseus a bag that contained all of the winds that would keep him from home so he could get there as fast as possible. Somehow this was a thing and after staying there for a day, they headed home at rate of \( f(x) = -10x^2 + 200x -190 \) where \( f(x) \) is distance from Aeolia in miles and \( x \) is time in days.

1. What could the negative sign in front of the quadratic element indicate about their journey?
2. If Ithaca, their home, is 850 miles away, did they reach it? Explain how you came to this conclusion.
3. Solve the equation, and along with the information from question 1 and 2, interpret what the roots mean and describe what might have happened to the crew. When possible use specific numbers that you found on the graph to help your description be as precise as possible.

1. Students should recognize that a negative quadratic means that they will go away from Aeolia (the 0 point), but then will reach a stopping point and come back.
2. Students should complete the square and get \( 10(x-10)^2 = 810 \) and find that the vertex is (10, 810), which would mean that they got very close, but did not make it to Ithaca. Students should demonstrate that they made the decision to complete the square in order to find the vertex.
3. Students should solve and get roots of 1 and 19, which indicate the start and stop of their journey. This should inform students that the crew did not make it to Ithaca after 9 days of travel. The crew then came all the way back to Aeolia after another 9 day journey. Teacher should look out for confusion from students about the y-intercept and the vertex being at an x-value of 10. The problem discusses very briefly that they left after a day (root of 1) and this information should fix any confusion.

Actual Story: Odysseus and his men sailed home after Aeolus on a journey that should have taken 10 days. Odysseus, not trusting his men to be strong-willed enough to not open the bag out of curiosity, didn’t tell them what was in it and somehow stayed awake for 9 nights. On the tenth, he awoke to find out that, sure enough, his men had opened the bag and the bad winds were pushing them on their way back to Aeolus after being so close to Ithaca. This, like most of the bad things that happened in the Odyssey, probably could have been avoided. A simple, “Don’t open the bag or we won’t get home” probably would’ve worked.
Stop 4/5- Telepylos the land of the Laestrygonians/Aeaea the Island of Circe:

The next stop for the men is Telepylos, the land of the Laestrygonians. They are cannibals who eat people at a rate of $f(x) = 7x + 5$ where $f(x)$ is people and $x$ is time in minutes. The second is Aeaea the island of Circe, a Witch-God who turns men into pigs at a rate of $f(x) = x^2 + 3x$. As I’m sure you can tell, neither of these visits went very well for the men.

1. Draw both rates on graph paper. Using this, decide at what point Circe turns more men into pigs than the Laestrygonians ate.
2. Write a table for the two rates, then decide who was more destructive to Odysseus and his crew if they only stayed at each place for 3 minutes, then do the same for 6 minutes. Explain why this is the case and why this makes sense.
3. Find the time where the number of people Circe turns into pigs equals the number of men the Laestrygonians ate using only the two equations. Explain why you used the method you used.

1. Students should see that after $x = 5$, the number of men Circe turns into pigs is greater.
2. | $x$ | $f(x) = x^2 + 3x$ | $f(x) = 7x + 5$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
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<td>1</td>
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<tr>
<td>6</td>
<td>54</td>
<td>47</td>
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</table>

3 minutes- Laestrygonians, 6 minutes- Circe. Students should understand here that linear rates do not grow as fast in the long term as quadratic rates. This activity should also show them an alternative to graphing for viewing the information.

3. Students might have variety of responses here, look for their reasoning and what factors prompted them to use a certain method. This activity serves to give students three ways to represent information given to them and be able to interpret all of them in a context.

Actual Story: The Laestrygonians ended up eating a lot of people and even smashed 11 of Odysseus’ ships. They were bad news and the men spent very little time on their island. Circe, however, ended up being nice and feeding the whole crew (after she turned them all into pigs and Odysseus came to the rescue). They stayed on Aeaea for a year to rest up, as you can imagine, getting eaten twice and pillaging is pretty tiring.
Who Shoots, Who Scores?

Dave Segall

Introduction: A free throw is a type of shot made in basketball where a player tries to score a point from the foul line undefended by their opponent. This lesson has students simulate free throw attempts to construct an understanding of foundational vocabulary and concepts in probability theory. Students will use the context of shooting free throws in basketball to build upon their understanding of probability by formalizing the terminology used to describe probability models. In performing a simulation of taking free throws, students will connect the events of their simulation to the following vocabulary terms: sample space, subset, event, union, intersection, and complement. Throughout the activity, students answer questions relating to the results of their simulations, and their responses are ultimately generalized outside the context of basketball. The physical activity of their simulation keeps students engaged in the exploration process while the context of basketball provides a real-world situation in which the mathematical terminology of probability can be applied. Students will: get practice modelling with mathematics by describing the results of their simulations, make use of structure by connecting the context to the theory, and reason abstractly/quantitatively by extending their results to a general setting.

Course: Math II

NCSCOS:

- **NC.M2.S-CP.2** - Describe events as subsets of the outcomes in a sample space using characteristics of the outcomes or as unions, intersections, and complements of other events.
Objectives: As a result of this lesson, students will be able to use formal probability terminology to describe the possible outcomes associated with uncertain events and apply the following vocabulary in real-world contexts: *sample space, subset, event, union, intersection, complement.*

Learning Activities:
The main activity of this lesson is divided into three stages
1. Teacher Model
   - Introduce the activity and goal by explaining to the class that teacher will shoot two “free throw” shots by trying to throw a crumbled paper ball into a basket, with the intent of being able to describe this scenario using the language of probability
   - Hand out slips of paper with lesson Focus Questions: *(Resource 1)*
     i. What are the possible outcomes when teacher shoots 2 free throws?
     ii. Make a guess: how many shots do you think teacher will make?
   - Have students do Think-Pair-Share with Focus Question’s above (give them a minute to answer the questions on their own, then spend a minute talking about their answer with a partner)
   - Display the table below to class and survey the class to see how many students think teacher will make one, both, or neither free throws. Record the number of students for each

<table>
<thead>
<tr>
<th>Outcome/Event</th>
<th>Make 1 Shot</th>
<th>Make Both</th>
<th>Make None</th>
</tr>
</thead>
<tbody>
<tr>
<td># Students</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

   - Shoot 2 free throws and record the outcome. Use results to introduce vocab terms *sample space* and *event*, connecting the table on the board and these vocab terms

2. Students Do Free Throws
   - Handout a copy of “Free Throw Scoreboard” *(Resource 2)* to each student
   - Go over directions of activity with students and have them complete activity in pairs. Will likely need to have students move desks/chairs out of the way or some pairs may need to do activity in hallway
   - When students have finished their 3 rounds of simulation, have them report their summary counts to teacher

3. Regroup, Closure, and Formative Assessment
   - Once all pairs are done with their simulation and all class data is recorded by teacher, go over the questions that were present on Resource 2 as a class by asking a pair to share their results for that round and how they answered the question
   - Each of the three questions relates to the terms *complement, union, or intersection*, so create a visual (Venn Diagram) for one group’s results to illustrate these vocab terms, connecting the question answer to the definition (two of the questions ask for visual representations, so make sure to indicate that the Venn Diagram can be used for these questions and ask a student to explain how it conveys the information for which the question asked)
   - Having students report their score summaries to teacher after each round is a way of tracking student progress throughout the activity by holding students accountable for their work in the activity. Further, the check-ins can be used to ask students probing questions
related to the questions asked in the activity, and it affords students the opportunity to ask teacher any questions as they come up throughout the activity

- After discussing ideas as a class, have students complete “Post Game Assessment” (Resource 3) as a formative assessment, making sure to collect from students afterwards

**Materials and Resources:**

The necessary materials are the following

- Paper for students to crumble up into paper balls – probably best to do ahead of time so that students don’t get carried away. Need one “ball” per pair
- Baskets/Buckets/Containers of some kind to shoot free throws into and each pair should have their own “hoop”, should aim to have them all the same size if possible
- Handouts (Resources 1-3) preprinted to give to students

**Resource 1:**

*Think-Pair-Share* - Take a minute to think the following two questions regarding the situation just described and write down your thoughts. After a minute, share your thoughts with the person next to you for a minute.

1. What are the possible things that can happen when teacher takes two free throws?

2. What do you think will happen? How many shots will teacher make?
Free Throw Score Board (Resource 2)

Directions: We will be doing this activity in pairs, so everybody needs a partner. Everybody will complete three rounds of the three free throw shots. In each round, record whether you made the free throw or not for each of your three shots. Record your partner’s results as well. After a round, record the total number of shots you and your partner made combined. For example, if on the first round I make one shot while my partner makes three, our total for that round is four shots made. Each round is followed by a question, so answer the questions as you go. After finishing each round, report your results for the round to the teacher to check-in. You should take shots about seven feet from your basket.

Round ONE:

<table>
<thead>
<tr>
<th>Partner (Names)</th>
<th>Shot 1</th>
<th>Shot 2</th>
<th>Shot 3</th>
<th>Number Made</th>
</tr>
</thead>
<tbody>
<tr>
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Total Number Made:__________________

Q1: In preparing for this activity, we decided that possible outcomes for the teacher trial were making one shot, making two shots, or making no shots. What are the possible outcomes (number made) that can happen when you take three shots? Which of these possible outcomes did not happen between you and your partner? We call this set of outcomes the complement.

Round TWO:

<table>
<thead>
<tr>
<th>Partner (Names)</th>
<th>Shot 1</th>
<th>Shot 2</th>
<th>Shot 3</th>
<th>Number Made</th>
</tr>
</thead>
<tbody>
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Total Number Made:__________________
Q2: Look back at your shooting outcomes from round one and your list of all possible outcomes. Which of the possible outcomes (number made) occurred in either round one or round two? We call this set of outcomes the union of rounds one and two.

Round THREE:

<table>
<thead>
<tr>
<th>Partner (Names)</th>
<th>Shot 1</th>
<th>Shot 2</th>
<th>Shot 3</th>
<th>Number Made</th>
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Total Number Made: _______________

Q3: Again, review your outcomes (number made) across all three rounds. Are there any outcomes which occurred on all three rounds? Which outcomes were they? We call this set of outcomes the intersection of rounds one, two, and three.

Using your results from above, identify the following sets of outcomes:

Q4: What is the complement of the outcomes from round 3? Explain why.

Q5: What is the intersection of rounds one and two? Can you think of any way to represent the outcomes in rounds one and two as well as their intersection visually? Sketch something!

Q6: What is the union of rounds 2 and 3? Can you incorporate all of the information from all questions into one visual representation? Try something!
Post-Game Assessment (Resource 3)

Choose one of the following four situations to answer questions about:

1. **Cookout:** You are going to cookout and know that you want to get a tray, but you haven’t decided exactly what to get yet. With the help of a friend, you’ve narrowed down your options. For the entrée, its between two hotdogs or a big double burger. For your sides, its between fries, chicken nuggets, or doubling up on either. Lastly, the drink is between cheerwine or a chocolate shake.

2. **Fortnite:** You and 3 friends are playing Fortnite, and you’ve worked together so that the 4 of you are in the last 5 players. Unfortunately, you and your friends turn on each other, and the 5th player (who is a stranger) is guaranteed to win. One of you will come in 5th, another in 4th, another in 3rd, and the remaining person will come in 2nd when you lose to the stranger.

3. **Phones:** You are getting a new phone and setting up a new plan with a service provider. You are choosing between Sprint, Verizon, and AT&T. You also must decide what phone to get. You are choosing between the new iPhoneXR or the new Galaxy S9.

   a.) Describe the sample space of your chosen situation. Use clear notation and include all possible outcomes.

   b.) Choose your favorite of all the outcomes in your chosen situation. Come up with two subsets of events whose intersection is your chosen outcome.

   c.) Explain the difference between the intersection of events and the union of events by using an example from your chosen situation. Make it clear how your example demonstrates the difference between these two ideas by using words or drawing a picture.

   d.) Choose your least favorite outcome from your situation’s sample space. What is the complement of this outcome? How do you know?